

Anomalous Higgs couplings in angular asymmetries of $H \rightarrow Z \ell^+ \ell^-$ and $e^+ e^- \rightarrow HZ$

Martin Beneke^a, Diogo Boito^a, and Yu-Ming Wang^{a,b}

^a*Physik Department T31, Technische Universität München,
James-Frank-Straße 1, D-85748 Garching, Germany*

^b*Institut für Theoretische Teilchenphysik und Kosmologie, RWTH Aachen
University, D-52056 Aachen, Germany*

Abstract

We study in detail the impact of anomalous Higgs couplings in angular asymmetries of the crossing-symmetric processes $H \rightarrow Z \ell^+ \ell^-$ and $e^+ e^- \rightarrow HZ$. Beyond Standard Model physics is parametrized in terms of the $SU(3) \times SU(2)_L \times U(1)_Y$ dimension-six effective Lagrangian. In the light of present bounds on $d = 6$ interactions we study how angular asymmetries can reveal non-standard CP-even and CP-odd couplings. We provide approximate expressions to all observables of interest making transparent their dominant dependence on anomalous couplings. We show that some asymmetries may reveal BSM effects that are hidden in other observables. In particular, CP-even and CP-odd $d = 6$ $HZ\gamma$ couplings as well as (to a lesser extent) $HZ\ell^+\ell^-$ contact interactions can generate asymmetries at the several percent level, while having small or no effects on the di-lepton invariant mass spectrum of $H \rightarrow Z \ell^+ \ell^-$. Finally, the higher di-lepton invariant mass probed in $e^+ e^- \rightarrow HZ$ leads to interesting differences in the asymmetries with respect to those of $H \rightarrow Z \ell^+ \ell^-$ that may lead to complementary anomalous coupling searches at the LHC and $e^+ e^-$ colliders.

1 Introduction

The discovery of a light boson H with mass around 125 GeV in the first run of LHC [1, 2] opened a new window to physics beyond the Standard Model (BSM). At present, the Standard Model (SM) Higgs quantum numbers, $J^P = 0^+$, are favoured by the data — all other tested hypotheses have been excluded at confidence levels above 95% [3, 4]. Furthermore, from the study of the signal strengths of the new state, all Higgs couplings to SM particles are compatible with SM predictions (see e.g. Ref. [5] and references therein). In particular, those to the W and Z bosons are constrained to be within 10 % of their SM values [6]. In the absence of evidence for any other new state, the SM seems to be a good effective field theory (EFT) above the electroweak scale, at least up to the scales currently probed by the LHC.

In the spirit of an EFT — assuming the characteristic scale Λ of BSM physics to be much larger than the electroweak scale — the SM should be supplemented with

all operators compatible with its symmetries. The $SU(2)_L \times U(1)_Y$ symmetry can be linearly or non-linearly realized. The non-linear realization gives rise to a theory in close analogy to ChPT [7–12], with an additional light, SM-singlet scalar. In the linear realization, that we adopt in this work, one must add to the SM all dimension-six operators constructed from the SM fields [13, 14]. These operators are suppressed by the large scale Λ and generate anomalous couplings of the Higgs boson.

The present status of the search for BSM physics in the Higgs sector is somewhat similar to that of the flavour sector of the SM, in which evidence for BSM physics has proven to be much more elusive than naively expected. In the search for new flavour-changing neutral currents, dedicated observables constructed from the angular distribution of $B \rightarrow K^* \ell^+ \ell^-$ have been constructed to unveil BSM effects in weak interactions more efficiently [15, 16]. The angular distribution of the analogous decay $H \rightarrow Z \ell^+ \ell^-$ offers similar possibilities.

The study of $H \rightarrow Z \ell^+ \ell^-$, with the on-shell Z also decaying into $\ell^+ \ell^-$, has a long history. Angular distributions were used in the determination of the Higgs quantum numbers [3, 4] as suggested years ago (see e.g. Refs. [17–20]). More recently, the di-lepton-mass distribution has been proposed as a way to reveal effects that would otherwise be hidden in the total decay width [21–23]. The full angular distribution of the final state leptons has been revisited recently [24] in the framework of the EFT parametrization of anomalous couplings, and it was shown that angular asymmetries can be constructed in order to reveal effects of anomalous Higgs couplings that would remain hidden even in the di-lepton invariant mass distribution.

In the present work we perform an extended study of the angular asymmetries of $H \rightarrow Z(\rightarrow \ell^+ \ell^-) \ell^+ \ell^-$ and of the crossing-symmetric reaction $e^+ e^- \rightarrow H Z(\rightarrow \ell^+ \ell^-)$. The latter process should be measured with high precision at a high-energy $e^+ e^-$ collider (such as the ILC [25]) and provide a clean way to extract the Higgs couplings [26–28]. In the massless lepton limit, the two processes are described by the same set of six form factors, albeit in different kinematic regimes, related by analyticity. The form factors can be written in terms of the couplings of the general $d = 6$ Lagrangian. Ignoring loop corrections and neglecting the lepton masses, the processes are described by six independent angular functions of the three independent angles among the four leptons, which can be expressed in terms of the six form factors. Our focus is on these asymmetries, their sensitivity to anomalous Higgs couplings, and the interplay between the asymmetries and the di-lepton mass distributions.

In $H \rightarrow Z \ell^+ \ell^-$, we show that the most promising anomalous coupling that could generate sizeable asymmetries is the $d = 6$ $H Z \gamma$ interaction. The contact interactions $H Z \ell^+ \ell^-$, whose effects were recently investigated in Ref. [24], also have a more prominent impact in the asymmetries than in the decay rate. However, the present constraints on these couplings make the magnitude of the asymmetries rather small. Next, we perform a study of the total cross section at intermediate energies for the reaction $e^+ e^- \rightarrow H Z(\rightarrow \ell^+ \ell^-)$ and of angular asymmetries akin to those of $H \rightarrow Z \ell^+ \ell^-$. We fully exploit the crossing-symmetric nature of these processes to make the relation between them transparent. Although described by the same form factors, the sensitivity to specific BSM couplings differs in the asymmetries of these two reactions due to the different characteristic energy scale involved. In $H \rightarrow Z \ell^+ \ell^-$ the $d = 6$ corrections scale as m_H^2/Λ^2 while in $e^+ e^- \rightarrow H Z$ they scale as q^2/Λ^2 , where q^2 , the center-of-mass (CM) energy of the $e^+ e^-$ pair, must be larger than $(217 \text{ GeV})^2$ to produce the $H Z$ final

state on-shell.

At present, the experimental study of the di-lepton mass distribution and of the angular asymmetries in $H \rightarrow Z(\rightarrow \ell^+\ell^-)\ell^+\ell^-$ is not feasible due to low statistics — ATLAS [29] and CMS [4] observed only around 30 $H \rightarrow ZZ^* \rightarrow 4\ell$ events each. However, higher luminosities will permit these studies in the future. With an integrated luminosity of 350 fb^{-1} at 14 TeV, which could be reached by 2021 [30, 31], the number of observed events could attain 1000. With the high-luminosity up-grade (integrated luminosity 3000 fb^{-1}), this number was estimated by a recent study of the sensitivity to anomalous Higgs-gauge boson interactions to be of the order of 6000 [32]. The number of reconstructed events of $e^+e^- \rightarrow HZ(\rightarrow \ell^+\ell^-)$ at an e^+e^- collider would be approximately 2000 at $\sqrt{q^2} = 250 \text{ GeV}$ with an integrated luminosity of 250 fb^{-1} [32].

The paper is organized as follows. In Sec. 2 we discuss the relevant operators in the linear realization of the $d = 6$ Lagrangian and the relevant Higgs anomalous couplings. In Sec. 3 we study the angular distribution of $H \rightarrow Z\ell^+\ell^-$ and we show some of the promising angular asymmetries. In Sec. 4 we perform a similar study of the reaction $e^+e^- \rightarrow HZ(\rightarrow \ell^+\ell^-)$. Our calculations are done at tree level. However, in Sec. 5 we discuss briefly the generic effect of SM loops. We summarize in Sec. 6. We relegate to App. A the kinematics and definitions of angular distributions, while the explicit expressions of the angular coefficient functions are given in App. B.

2 Effective Lagrangian and couplings

In order to parametrize BSM effects in a general way, we resort to the linear realization of the $SU(2)_L \times U(1)_Y$ SM electroweak symmetry. Assuming the new physics sector to be characterized by a scale Λ , larger than the electroweak scale, the SM is supplemented with 59 independent $d = 6$ operators [13, 14]. This Lagrangian can be schematically cast as

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}}^{(4)} + \frac{1}{\Lambda^2} \sum_{k=1}^{59} \alpha_k \mathcal{O}_k, \quad (1)$$

where the α_k is the coupling of operator \mathcal{O}_k . The effective Lagrangian implies a parametrization of anomalous Higgs interactions (contained in \mathcal{O}_k) constrained by the SM gauge symmetry. In our expressions, we often employ the notation $\hat{\alpha}_k$ defined as

$$\hat{\alpha}_k = \frac{v^2}{\Lambda^2} \alpha_k, \quad (2)$$

where v is the classical Higgs vacuum expectation value. The dimensionless coefficients $\hat{\alpha}_k$ should be smaller than $\mathcal{O}(1)$ for the EFT description to be applicable.

Different choices for the operator basis are possible and in use. Here we stick to the basis defined in Ref. [14]. In practice we only need to work with a subset of the 59 operators, since not all of them contribute at tree level to the processes of interest. Furthermore, assuming minimal-flavour violation to avoid tree-level flavour-changing neutral currents, flavour matrices of operators that involve a left-handed doublet and a right-handed singlet are fixed to be the same as in the SM Yukawa couplings. Within this approximation, these operators are proportional to lepton masses and are henceforward neglected.¹

¹In $H \rightarrow Z\ell^+\ell^-$ the lepton mass corrections are at most of the order of $m_\tau^2/m_H^2 \approx 2 \times 10^{-4}$. The

Table 1: The subset of $d = 6$ operators that contribute to $H \rightarrow Z\ell^+\ell^-$ and $e^+e^- \rightarrow HZ$ in the basis defined in Ref. [14]. The four-lepton operator given in Eq. (8) gives an indirect contribution solely through the redefinition of δ_{GF} and is not listed in this table.

$\Phi^4 D^2$	$X^2 \Phi^2$	$\psi^2 \Phi^2 D$
$\mathcal{O}_{\Phi\Box} = (\Phi^\dagger\Phi)\Box(\Phi^\dagger\Phi)$	$\mathcal{O}_{\Phi W} = (\Phi^\dagger\Phi)W_{\mu\nu}^I W^{I\mu\nu}$	$\mathcal{O}_{\Phi\ell}^{(1)} = (\Phi^\dagger i\overleftrightarrow{D}_\mu\Phi)(\bar{\ell}\gamma^\mu\ell)$
$\mathcal{O}_{\Phi D} = (\Phi^\dagger D^\mu\Phi)^*(\Phi^\dagger D_\mu\Phi)$	$\mathcal{O}_{\Phi B} = (\Phi^\dagger\Phi)B_{\mu\nu}B^{\mu\nu}$	$\mathcal{O}_{\Phi\ell}^{(3)} = (\Phi^\dagger i\overleftrightarrow{D}_\mu^I\Phi)(\bar{\ell}\gamma^\mu\tau^I\ell)$
	$\mathcal{O}_{\Phi WB} = (\Phi^\dagger\tau^I\Phi)W_{\mu\nu}^I B^{\mu\nu}$	$\mathcal{O}_{\Phi e} = (\Phi^\dagger i\overleftrightarrow{D}_\mu\Phi)(\bar{e}\gamma^\mu e)$
	$\mathcal{O}_{\Phi\widetilde{W}} = (\Phi^\dagger\Phi)\widetilde{W}_{\mu\nu}^I W^{I\mu\nu}$	
	$\mathcal{O}_{\Phi\widetilde{B}} = (\Phi^\dagger\Phi)\widetilde{B}_{\mu\nu}B^{\mu\nu}$	
	$\mathcal{O}_{\Phi\widetilde{WB}} = (\Phi^\dagger\tau^I\Phi)\widetilde{W}_{\mu\nu}^I B^{\mu\nu}$	

The operators considered in this work are listed in Tab. 1. The notation and conventions follow those of Ref. [33]. The Higgs doublet is denoted Φ . The field strength tensors for the $SU(2)_L \times U(1)_Y$ gauge group are

$$\begin{aligned} W_{\mu\nu}^I &= \partial_\mu W_\nu^I - \partial_\nu W_\mu^I - g\epsilon^{IJK}W_\mu^J W_\nu^K, & I = 1, 2, 3, \\ B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu, \end{aligned} \quad (3)$$

with the gauge couplings g and g' , respectively. With a tilde we denote the dual field strength tensors

$$\widetilde{X}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}X^{\rho\sigma} \quad (4)$$

where $X = W^I, B$ and $\epsilon_{0123} = +1$. When acting on $SU(2)$ doublets, the covariant derivative is written

$$D_\mu = \partial_\mu + ig\frac{\tau^I}{2}W_\mu^I + ig'YB_\mu, \quad (5)$$

where Y is the hypercharge and τ^I are the Pauli matrices. In Tab. 1 the left-handed lepton doublets and the right-handed charged leptons are written ℓ_p^i and e_p , where $i = 1, 2$ and $p = 1, 2, 3$ are weak-isospin and flavour indices, respectively. We make the simplifying assumption, stronger than minimal flavour violation, that the coefficients $\alpha_{\Phi\ell}^{(1)}$, $\alpha_{\Phi\ell}^{(3)}$, and $\alpha_{\Phi e}$ are flavour independent, and define the fermion-bilinear operators with flavour indices contracted. Because of this the operators are hermitian and all couplings α_k are real. A few comments on the operators in Tab. 1 are in order.

- The $\psi^2 \Phi^2 D$ type operators yield contact $HZ\ell\ell$ interactions as well as modifications of the gauge-boson couplings to leptons.
- The $X^2\Phi^2$ operators generate anomalous couplings of the Higgs to ZZ , γZ , and WW . After performing field redefinitions of the gauge fields, the SM Higgs couplings to gauge bosons are not modified due to cancellations against the redefinitions of input parameters (see Ref. [34] and Eq. (6) below).

typical contribution from a $d = 6$ operator scales as m_H^2/Λ^2 which is 5-10 times larger for Λ of a few TeV.

- Operators of the type $\Phi^4 D^2$ modify the Higgs-gauge couplings and entail a redefinition of the Higgs field to preserve canonically normalized kinetic terms.

The $d = 4$ couplings of the electroweak sector of the SM Lagrangian are the gauge couplings g , g' , the Higgs self-coupling λ , and the classical Higgs vacuum expectation value v . We trade these couplings for the experimental observables G_F (the Fermi constant as measured in $\mu \rightarrow e \nu_\mu \bar{\nu}_e$ decay), the Z mass m_Z , the electromagnetic coupling α_{em} , and the Higgs mass m_H . In the presence of $d = 6$ operators, the first three of these quantities are given by

$$m_Z = m_{Z_0} (1 + \delta_Z), \quad G_F = G_{F_0} (1 + \delta_{G_F}), \quad \alpha_{\text{em}} = \alpha_{\text{em}_0} (1 + \delta_A), \quad (6)$$

where X_0 denotes the quantity X in the absence of $d = 6$ operators, expressed in terms of the Lagrangian parameters g , g' , and v . The above relations are then inverted to express the g , g' , and v in terms of m_Z , G_F and α_{em} and the $d = 6$ couplings. The explicit expressions for the $d = 6$ contributions to Eq. (6) in our basis read [33, 35, 36]

$$\delta_Z = \hat{\alpha}_{ZZ} + \frac{1}{4} \hat{\alpha}_{\Phi D}, \quad \delta_{G_F} = -\hat{\alpha}_{4L} + 2\hat{\alpha}_{\Phi\ell}^{(3)}, \quad \delta_A = 2\hat{\alpha}_{AA}. \quad (7)$$

The combinations of coupling coefficients α_{ZZ} and α_{AA} are defined in Eq. (11) below. In δ_{G_F} a four-lepton operator (not listed in the Tab. 1) intervenes

$$\mathcal{O}_{4L}^{prst} = (\bar{\ell}_p \gamma_\mu \ell_r) (\bar{\ell}_s \gamma^\mu \ell_t), \quad (8)$$

with p , r , s , and t denoting flavour indices. We assume that the coefficients of \mathcal{O}_{4L}^{prst} are flavour independent. In the expressions below we will also use the Weinberg angle

$$\sin^2 \theta_W \equiv s_W^2 = \frac{1}{2} \left(1 - \sqrt{1 - \frac{2\sqrt{2}\pi\alpha_{\text{em}}}{m_Z^2 G_F}} \right), \quad \cos^2 \theta_W \equiv c_W^2 = 1 - s_W^2. \quad (9)$$

It should be understood as an abbreviation for the combination of input parameters as given, which appears after eliminating the $d = 4$ Lagrangian couplings as described above.

Apart from the SM tree contributions we only consider effects of order $1/\Lambda^2$ on the decay amplitude. In the broken-symmetry phase the effective Lagrangian Eq. (1) generates the terms

$$\begin{aligned} \mathcal{L}_{\text{eff}} \supset & c_{ZZ}^{(1)} H Z_\mu Z^\mu + c_{ZZ}^{(2)} H Z_{\mu\nu} Z^{\mu\nu} + c_{Z\tilde{Z}} H Z_{\mu\nu} \tilde{Z}^{\mu\nu} + c_{AZ} H Z_{\mu\nu} A^{\mu\nu} + c_{A\tilde{Z}} H Z_{\mu\nu} \tilde{A}^{\mu\nu} \\ & + H Z_\mu \bar{\ell} \gamma^\mu (c_V + c_A \gamma_5) \ell + Z_\mu \bar{\ell} \gamma^\mu (g_V - g_A \gamma_5) \ell - g_{\text{em}} Q_\ell A_\mu \bar{\ell} \gamma^\mu \ell, \end{aligned} \quad (10)$$

which include the relevant tree-level SM terms. We omit the $H\gamma\gamma$ vertex, since it does not contribute to the processes studied here within our approximations. The effective couplings of this Lagrangian are related to the coefficients α_k of the fundamental $d = 6$ operators as given explicitly in Tab. 1. We define the following combinations of coupling coefficients:

$$\alpha_{ZZ}^{(1)} = \alpha_{\Phi\Box} - \frac{1}{2} \delta_{G_F} + \frac{1}{4} \alpha_{\Phi D},$$

$$\begin{aligned}
\alpha_{ZZ} &= c_W^2 \alpha_{\Phi W} + s_W^2 \alpha_{\Phi B} + s_W c_W \alpha_{\Phi WB}, \\
\alpha_{AZ} &= 2s_W c_W (\alpha_{\Phi W} - \alpha_{\Phi B}) + (s_W^2 - c_W^2) \alpha_{\Phi WB}, \\
\alpha_{AA} &= s_W^2 \alpha_{\Phi W} + c_W^2 \alpha_{\Phi B} - s_W c_W \alpha_{\Phi WB}.
\end{aligned} \tag{11}$$

with analogous expressions for $\alpha_{Z\tilde{Z}}$ and $\alpha_{A\tilde{Z}}$, where the couplings on the r.h.s. are replaced by their tilde counterparts. The Higgs-gauge couplings of Eq. (10) are then given by

$$\begin{aligned}
c_{ZZ}^{(1)} &= m_Z^2 (\sqrt{2} G_F)^{1/2} \left(1 + \hat{\alpha}_{ZZ}^{(1)} \right), \\
c_{ZZ}^{(2)} &= (\sqrt{2} G_F)^{1/2} \hat{\alpha}_{ZZ}, \\
c_{Z\tilde{Z}} &= (\sqrt{2} G_F)^{1/2} \hat{\alpha}_{Z\tilde{Z}}, \\
c_{AZ} &= (\sqrt{2} G_F)^{1/2} \hat{\alpha}_{AZ}, \\
c_{A\tilde{Z}} &= (\sqrt{2} G_F)^{1/2} \hat{\alpha}_{A\tilde{Z}}.
\end{aligned} \tag{12}$$

The contact $HZ\ell\ell$ couplings can be written as

$$\begin{aligned}
c_V &= \sqrt{2} G_F m_Z \hat{\alpha}_{\Phi\ell}^V, \\
c_A &= \sqrt{2} G_F m_Z \hat{\alpha}_{\Phi\ell}^A.
\end{aligned} \tag{13}$$

with

$$\begin{aligned}
\hat{\alpha}_{\Phi\ell}^V &= \hat{\alpha}_{\Phi e} + \left(\hat{\alpha}_{\Phi\ell}^{(1)} + \hat{\alpha}_{\Phi\ell}^{(3)} \right), \\
\hat{\alpha}_{\Phi\ell}^A &= \hat{\alpha}_{\Phi e} - \left(\hat{\alpha}_{\Phi\ell}^{(1)} + \hat{\alpha}_{\Phi\ell}^{(3)} \right).
\end{aligned} \tag{14}$$

Note that in the operator basis employed here $\hat{\alpha}_{\Phi\ell}^V$ and $\hat{\alpha}_{\Phi\ell}^A$ are, in general, different. Therefore, in the effective Lagrangian we have both left-handed and right-handed $HZ\ell\ell$ couplings. This contrasts with the so-called SILH basis [37], where the absence of the operators $\mathcal{O}_{\Phi\ell}^{(1,3)}$ implies $\hat{\alpha}_{\Phi\ell}^V = \hat{\alpha}_{\Phi\ell}^A$ and hence only right-handed $HZ\ell\ell$ couplings enter the effective Lagrangian directly [38].

The same combinations $\hat{\alpha}_{\Phi\ell}^{V,A}$ also participate in the $d = 6$ corrections to the Z couplings to fermions in Eq. (10), that can be cast as

$$\begin{aligned}
g_V &= \frac{m_Z}{2} (\sqrt{2} G_F)^{1/2} \left[(1 - 4s_W^2) - \delta g_V \right], \\
g_A &= \frac{m_Z}{2} (\sqrt{2} G_F)^{1/2} (1 + \delta g_A).
\end{aligned} \tag{15}$$

The corrections $\delta g_{V,A}$ from the $d = 6$ operators are given by

$$\begin{aligned}
\delta g_V &= -\hat{\alpha}_{\Phi\ell}^V + \frac{\hat{\alpha}_{\Phi D}}{4} + \frac{\delta_{G_F}}{2} + \frac{4s_W^2}{c_{2W}} \left[\frac{\hat{\alpha}_{\Phi D}}{4} + \frac{c_W}{s_W} \hat{\alpha}_{\Phi WB} + \frac{\delta_{G_F}}{2} \right], \\
\delta g_A &= -\hat{\alpha}_{\Phi\ell}^A - \frac{\hat{\alpha}_{\Phi D}}{4} - \frac{\delta_{G_F}}{2},
\end{aligned} \tag{16}$$

with $c_{2W} \equiv \cos 2\theta_W$. The contributions from $\hat{\alpha}_{\Phi D}$, $\hat{\alpha}_{\Phi WB}$, and δ_{G_F} arise from the redefinition of SM fields and the rewriting of Lagrangian parameters in terms of input parameters in the presence of $d = 6$ operators.

In this work, we are interested in the effects of Higgs anomalous couplings in certain angular asymmetries of $H \rightarrow Z\ell^+\ell^-$ and $e^+e^- \rightarrow HZ$. In order to estimate the maximal effect that is still possible, we have to incorporate the constraints on the anomalous couplings from all existing data, not necessarily related to Higgs observables. However, to the best of our knowledge, a full analysis along the lines of Refs. [38, 39] is not available in the operator basis employed here. Although different bases are related by a linear transformation, the results cannot be straightforwardly translated, since the correlations are not known (to us). We are particularly interested in the coefficients $\hat{\alpha}_{\Phi\ell}^{V,A}$ of the contact interactions. In the absence of a complete analysis, we perform here an order-of-magnitude estimate of the present constraints on $\hat{\alpha}_{\Phi\ell}^{V,A}$, which is sufficient for the discussion of the angular asymmetries.

In order to use data for $g_{V,A}$ to constrain $\hat{\alpha}_{\Phi\ell}^{V,A}$ one needs to estimate the allowed range for the other three combinations of Wilson coefficients that enter Eq. (16), namely, $\hat{\alpha}_{\Phi D}$, $\hat{\alpha}_{\Phi WB}$, and δ_{GF} . One therefore needs five observables. Apart from the Z coupling to leptons, g_V and g_A , we employ the electroweak precision observables S and T , and the W mass. The operators $\mathcal{O}_{\Phi WB}$ and $\mathcal{O}_{\Phi D}$ give tree-level contributions to S and T , respectively (for the explicit expressions in our basis, see [36]). Experimental values for these two parameters [40] constrain $\hat{\alpha}_{\Phi WB}$ and $\hat{\alpha}_{\Phi D}$ to be at the permille level. With these bounds as input, m_W can be utilized to constrain δ_{GF} , since

$$m_W = m_Z(1 - s_W^2)^{1/2} \left[1 - \frac{1}{2c_{2W}} \left(\frac{c_W^2}{2} \hat{\alpha}_{\Phi D} + s_W^2 \delta_{GF} + s_{2W} \hat{\alpha}_{\Phi WB} \right) \right], \quad (17)$$

with $s_{2W} \equiv \sin 2\theta_W$. This constrains δ_{GF} to be at the level of a few 10^{-3} . Finally, using these results and the tight constraints on $\delta g_{V,A}$ [41, 42], we find that $\hat{\alpha}_{\Phi\ell}^A$ and $\hat{\alpha}_{\Phi\ell}^V$ cannot exceed a few times 10^{-3} .² This agrees with the conclusion of Ref. [38] that the bounds on $\hat{\alpha}_{\Phi\ell}^{V,A}$ are at the permille level, though this refers to the SILH basis in which $\hat{\alpha}_{\Phi\ell}^A = \hat{\alpha}_{\Phi\ell}^V$.

Since we do not perform a full fit, we allow $\hat{\alpha}_{\Phi\ell}^A$ and $\hat{\alpha}_{\Phi\ell}^V$ to vary within slightly weaker bounds than those arising from the five observables described above. We therefore employ the conservative interval

$$\hat{\alpha}_{\Phi\ell}^{V,A} \in [-5, 5] \times 10^{-3}. \quad (18)$$

According to Eq. (2), for $\alpha_{\Phi\ell}^{V,A} = 1$, this corresponds to the BSM physics scale of $\Lambda \approx 3.5$ TeV. Note that the maximally allowed value of $\hat{\alpha}_{\Phi\ell}^V$ is smaller by a factor of about 4 compared to the one used in Ref. [24].

The $d = 6$ anomalous $HZ\gamma$ vertex also plays an important role in our analysis. This coupling is constrained by Higgs measurements, especially the direct searches for $H \rightarrow Z\gamma$ decays, which presently limit the branching fraction to about ten times the SM expectation [44, 45]. This leads to a weaker bound than those on $\hat{\alpha}_{\Phi\ell}^{V,A}$ and here we employ the result of Ref. [38], which with our definitions reads

$$\hat{\alpha}_{AZ} \in [-1.3, 2.6] \times 10^{-2}. \quad (19)$$

²The triple-gauge boson coupling Δg_1^Z [38, 43] could also be used to constrain δ_{GF} but leads to less stringent bounds than the value of m_W does. Data for the decay $Z \rightarrow \bar{\nu}\nu$ help disentangling $\hat{\alpha}_{\Phi\ell}^{(1)}$ and $\hat{\alpha}_{\Phi\ell}^{(3)}$ but this is immaterial to the present work.

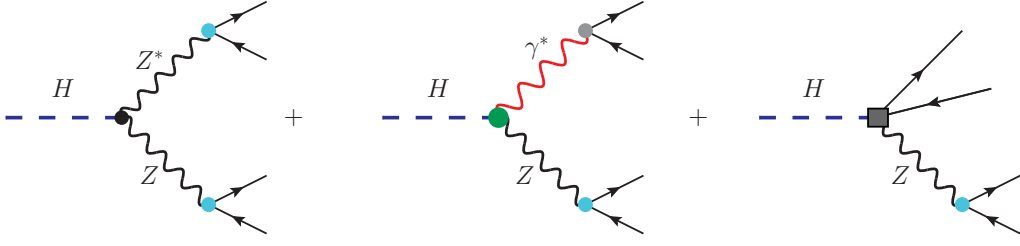


Figure 1: Feynman diagrams for the decay $H \rightarrow Z(\rightarrow \ell^+\ell^-)\ell^+\ell^-$.

One-loop corrections to the SM amplitude give contributions to the $H \rightarrow Z\ell^+\ell^-$ and $e^+e^- \rightarrow HZ$ processes studied in this paper that can be of the same order of $d = 6$ terms. They have been computed in the past [46–49] and should eventually be included in a quantitative extraction of the anomalous couplings from data. Since this data is not yet available, and our purpose is to determine the sensitivity of angular observables to $d = 6$ operators, we neglect loop corrections here. In Sec. 5 we provide a rough estimate in order to ascertain whether or not loop effects affect the main conclusions of this work. Loop corrections to amplitudes with $d = 6$ operator insertions are evidently negligible.

In the following we discuss the effects of anomalous Higgs couplings of Eqs. (12) and (13) in the differential decay width of $H \rightarrow Z(\rightarrow \ell^+\ell^-)\ell^+\ell^-$, in the total cross section of $e^+e^- \rightarrow HZ(\rightarrow \ell^+\ell^-)$, as well as on angular asymmetries of these two crossing-symmetric processes. Two main scenarios will be investigated in detail. In the first we allow for non-vanishing $\hat{\alpha}_{\Phi\ell}^{V,A}$, which gives rise to the $HZ\ell\ell$ contact interaction of Eq. (13), and in the second for non-vanishing $\hat{\alpha}_{AZ}$. In each scenario we set the other couplings to zero. In scenarios with non-vanishing $\hat{\alpha}_{\Phi\ell}^{V,A}$ their contribution to $\delta g_{V,A}$ is taken into account.

3 Angular asymmetries of $H \rightarrow Z(\rightarrow \ell^+\ell^-)\ell^+\ell^-$

The decay of the on-shell Higgs boson to four leptons with an intermediate on-shell Z boson can proceed with an off-shell Z through the $H \rightarrow ZZ$ interaction, as in the SM, but with $d = 6$ operators added to the Lagrangian it can also proceed through a $HZ\gamma$ coupling or the contact interaction $HZ\ell\ell$. The three types of diagrams are depicted in Fig. 1.

3.1 Form factors and angular distribution

The amplitude for the decay $H(p_H) \rightarrow Z(p)(\rightarrow \ell^-(p_1)\ell^+(p_2))\ell^-(p_3)\ell^+(p_4)$ can be written as

$$\mathcal{M}(H \rightarrow Z(\rightarrow \ell^+\ell^-)\ell^+\ell^-) = \mathcal{M}_{HZ\ell\ell}^\mu \frac{1}{p^2 - m_Z^2 + i m_Z \Gamma_Z} \mathcal{M}_{Z\ell\ell,\mu}, \quad (20)$$

where \mathcal{M}_X^μ denotes the matrix element of process X with the polarization vector of the on-shell Z boson stripped off. As already mentioned, we neglect lepton mass effects. When squaring the amplitude we employ the narrow-width approximation for the

intermediate Z boson, but include spin correlations. Summing over spins of the final-state leptons, the four-fold differential decay width for the process $H \rightarrow Z(\rightarrow \ell^+ \ell^-) \ell^+ \ell^-$ in the massless lepton limit can be written as a function of the di-lepton invariant mass squared $q^2 = (p_3 + p_4)^2$ and of three angles (see App. A.1 for their definitions). The expression reads

$$\begin{aligned} \frac{d^4 \Gamma}{dq^2 d \cos \theta_1 d \cos \theta_2 d \phi} &= \frac{1}{2^{10} (2\pi)^5} \frac{1}{m_H^3} \frac{1}{m_Z \Gamma_Z} \lambda^{1/2}(m_H^2, m_Z^2, q^2) \sum_{\text{spins}} |\mathcal{M}_{HZ\ell\ell}^\mu \mathcal{M}_{Z\ell\ell,\mu}|^2 \\ &= \frac{1}{m_H} \mathcal{N}(q^2) \mathcal{J}(q^2, \theta_1, \theta_2, \phi). \end{aligned} \quad (21)$$

In the last equation we introduced the dimensionless function

$$\mathcal{J}(q^2, \theta_1, \theta_2, \phi) = \frac{1}{m_H^2} \sum_{\text{spins}} |\mathcal{M}_{HZ\ell\ell}^\mu \mathcal{M}_{Z\ell\ell,\mu}|^2, \quad (22)$$

and the normalization

$$\mathcal{N}(q^2) = \frac{1}{2^{10} (2\pi)^5} \frac{1}{\sqrt{r} \gamma_Z} \lambda^{1/2}(1, r, s), \quad (23)$$

written in terms of the dimensionless variables

$$s = \frac{q^2}{m_H^2}, \quad r = \frac{m_Z^2}{m_H^2} \approx 0.53, \quad \gamma_Z = \frac{\Gamma_Z}{m_H} \approx 0.020, \quad (24)$$

and the function $\lambda(a, b, c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc$. The maximum value of q^2 is $q_{\text{max}}^2 = (m_H - m_Z)^2 \approx (34.4 \text{ GeV})^2$ which gives

$$0 \leq s \leq \frac{(m_H - m_Z)^2}{m_H^2} \approx 0.075. \quad (25)$$

The decay of the on-shell Z boson is described by

$$\mathcal{M}_{Z\ell\ell}^\mu = \bar{u}(k_1, s_1) [\gamma^\mu (g_V - g_A \gamma_5)] v(k_2, s_2), \quad (26)$$

with the couplings given in Eq. (15). It is important to observe that g_A largely dominates the interaction $Z \rightarrow \ell^+ \ell^-$ due to the partial cancellation in the factor $(1 - 4s_W^2)$ in g_V . This fact plays an important role in the interpretation of the numerical results for the angular asymmetries.

Neglecting the lepton masses the general expression for the amplitude of $H \rightarrow Z(p) \ell^-(p_3) \ell^+(p_4)$ at $\mathcal{O}(1/\Lambda^2)$ in the $d = 6$ Lagrangian can be written in terms of six form factors [17, 18, 21, 23, 24]. Denoting them by $H_{i,V/A}$ ($i = 1, 2, 3$), we adopt the parametrization

$$\begin{aligned} \mathcal{M}_{HZ\ell\ell}^\mu &= \frac{1}{m_H} \bar{u}(p_3, s_3) \left[\gamma^\mu (H_{1,V} + H_{1,A} \gamma_5) + \frac{q^\mu \not{p}}{m_H^2} (H_{2,V} + H_{2,A} \gamma_5) \right. \\ &\quad \left. + \frac{\epsilon^{\mu\nu\sigma\rho} p_\nu q_\sigma}{m_H^2} \gamma_\rho (H_{3,V} + H_{3,A} \gamma_5) \right] v(p_4, s_4), \end{aligned} \quad (27)$$

where $\epsilon_{0123} = +1$ and $q = p_3 + p_4$. The form factors $H_{2,V/A}$ and $H_{3,V/A}$ vanish in the SM at tree level. The expressions for $H_{i,V/A}$ at $\mathcal{O}(1/\Lambda^2)$ are

$$\begin{aligned}
H_{1,V} &= -\frac{2m_H(\sqrt{2}G_F)^{1/2}r}{r-s}g_V\left(1+\hat{\alpha}_1^{\text{eff}}-\frac{\kappa}{r}\hat{\alpha}_{ZZ}-\frac{\kappa}{2r}\frac{Q_\ell g_{em}(r-s)}{s g_V}\hat{\alpha}_{AZ}\right), \\
H_{1,A} &= \frac{2m_H(\sqrt{2}G_F)^{1/2}r}{r-s}g_A\left(1+\hat{\alpha}_2^{\text{eff}}-\frac{\kappa}{r}\hat{\alpha}_{ZZ}\right), \\
H_{2,V} &= -\frac{2m_H(\sqrt{2}G_F)^{1/2}}{r-s}g_V\left[2\hat{\alpha}_{ZZ}+\frac{Q_\ell g_{em}(r-s)}{s g_V}\hat{\alpha}_{AZ}\right], \\
H_{2,A} &= \frac{4m_H(\sqrt{2}G_F)^{1/2}}{r-s}g_A\hat{\alpha}_{ZZ}, \\
H_{3,V} &= -\frac{2m_H(\sqrt{2}G_F)^{1/2}}{r-s}g_V\left[2\hat{\alpha}_{ZZ\tilde{Z}}+\frac{Q_\ell g_{em}(r-s)}{s g_V}\hat{\alpha}_{A\tilde{Z}}\right], \\
H_{3,A} &= \frac{4m_H(\sqrt{2}G_F)^{1/2}}{r-s}g_A\hat{\alpha}_{ZZ\tilde{Z}}, \tag{28}
\end{aligned}$$

where $Q_\ell = -1$. The couplings g_A and g_V are those of Eq. (15) including the $d = 6$ corrections. (Of course, within our approximations, this matters only when $g_{V,A}$ multiply the SM “1” in the bracket in $H_{1,V/A}$.) We defined the combinations

$$\begin{aligned}
\hat{\alpha}_1^{\text{eff}} &\equiv \hat{\alpha}_{ZZ}^{(1)} - \frac{m_H(\sqrt{2}G_F)^{1/2}(r-s)}{2\sqrt{r}}\frac{\hat{\alpha}_{\Phi l}^V}{g_V}, \\
\hat{\alpha}_2^{\text{eff}} &\equiv \hat{\alpha}_{ZZ}^{(1)} + \frac{m_H(\sqrt{2}G_F)^{1/2}(r-s)}{2\sqrt{r}}\frac{\hat{\alpha}_{\Phi l}^A}{g_A}, \tag{29}
\end{aligned}$$

where the couplings $\hat{\alpha}_{ZZ}^{(1)}$ and $\hat{\alpha}_{\Phi l}^{V/A}$ are defined in Eqs. (11) and (14), respectively. Last, we introduced

$$\kappa = 1 - r - s. \tag{30}$$

At order $1/\Lambda^2$, ignoring loop-suppressed contributions and lepton masses, the form factors of Eq. (28) are real. Note that the absence of i in front of the epsilon-symbol in Eq. (27) implies that with this definition real $H_{3,V/A}$ are CP-odd form factors as can also be seen from their expressions in Eq. (28).

Computing $\mathcal{J}(q^2, \theta_1, \theta_2, \phi)$ explicitly, we find nine independent angular structures with coefficient functions J_1, \dots, J_9 , which we write as³

$$\begin{aligned}
\mathcal{J}(q^2, \theta_1, \theta_2, \phi) &= J_1(1 + \cos^2 \theta_1 \cos^2 \theta_2 + \cos^2 \theta_1 + \cos^2 \theta_2) \\
&\quad + J_2 \sin^2 \theta_1 \sin^2 \theta_2 + J_3 \cos \theta_1 \cos \theta_2 \\
&\quad + (J_4 \sin \theta_1 \sin \theta_2 + J_5 \sin 2\theta_1 \sin 2\theta_2) \sin \phi
\end{aligned}$$

³ To make contact with Ref. [18], we remark that final-state interactions, which would generate (loop-suppressed) imaginary parts in the form factors, lead to six new angular structures. Denoting these new structures by $\delta\mathcal{J}$, the expression

$$\begin{aligned}
\delta\mathcal{J} &= (J_{10} \sin 2\theta_1 \sin \theta_2 + J_{11} \sin \theta_1 \sin 2\theta_2) \sin \phi + (J_{12} \sin 2\theta_1 \sin \theta_2 + J_{13} \sin \theta_1 \sin 2\theta_2) \cos \phi \\
&\quad + J_{14} \cos \theta_2(1 + \cos^2 \theta_1) + J_{15} \cos \theta_1(1 + \cos^2 \theta_2)
\end{aligned}$$

has to be added to Eq. (31). The new angular functions depend on the imaginary parts of the form factors $H_{i,V/A}$.

$$\begin{aligned}
& + (J_6 \sin \theta_1 \sin \theta_2 + J_7 \sin 2\theta_1 \sin 2\theta_2) \cos \phi \\
& + J_8 \sin^2 \theta_1 \sin^2 \theta_2 \sin 2\phi + J_9 \sin^2 \theta_1 \sin^2 \theta_2 \cos 2\phi. \quad (31)
\end{aligned}$$

The expressions for the non-vanishing J functions at $\mathcal{O}(1/\Lambda^2)$ in the limit $m_\ell \rightarrow 0$ in terms of the form factors of Eq. (27) are⁴

$$\begin{aligned}
J_1 &= 2 r s (g_A^2 + g_V^2) (|H_{1,V}|^2 + |H_{1,A}|^2), \\
J_2 &= \kappa (g_A^2 + g_V^2) [\kappa (|H_{1,V}|^2 + |H_{1,A}|^2) + \lambda \operatorname{Re} (H_{1,V} H_{2,V}^* + H_{1,A} H_{2,A}^*)], \\
J_3 &= 32 r s g_A g_V \operatorname{Re} (H_{1,V} H_{1,A}^*), \\
J_4 &= 4 \kappa \sqrt{r s \lambda} g_A g_V \operatorname{Re} (H_{1,V} H_{3,A}^* + H_{1,A} H_{3,V}^*), \\
J_5 &= \frac{1}{2} \kappa \sqrt{r s \lambda} (g_A^2 + g_V^2) \operatorname{Re} (H_{1,V} H_{3,V}^* + H_{1,A} H_{3,A}^*), \\
J_6 &= 4 \sqrt{r s} g_A g_V [4 \kappa \operatorname{Re} (H_{1,V} H_{1,A}^*) + \lambda \operatorname{Re} (H_{1,V} H_{2,A}^* + H_{1,A} H_{2,V}^*)], \\
J_7 &= \frac{1}{2} \sqrt{r s} (g_A^2 + g_V^2) [2 \kappa (|H_{1,V}|^2 + |H_{1,A}|^2) + \lambda \operatorname{Re} (H_{1,V} H_{2,V}^* + H_{1,A} H_{2,A}^*)], \\
J_8 &= 2 r s \sqrt{\lambda} (g_A^2 + g_V^2) \operatorname{Re} (H_{1,V} H_{3,V}^* + H_{1,A} H_{3,A}^*), \\
J_9 &= 2 r s (g_A^2 + g_V^2) (|H_{1,V}|^2 + |H_{1,A}|^2). \quad (32)
\end{aligned}$$

These expressions are valid beyond our approximations, where the $H_{i,V/A}$ form factors are all real. We used the notation $\lambda \equiv \lambda(1, r, s)$ and recall that $g_{V,A}$ implicitly contain $d = 6$ corrections, see Eq. (15). At order $\mathcal{O}(1/\Lambda^2)$, $H_{2,V/A}$ and $H_{3,V/A}$ contribute only through interference with the SM part of the form factors $H_{1,V/A}$. We drop the $\mathcal{O}(1/\Lambda^4)$ terms still contained in Eq. (32).

Only six of the functions J_i in Eq. (32) are independent. The following relations hold:

$$\begin{aligned}
J_5 &= \frac{\kappa}{4\sqrt{rs}} J_8, \\
J_7 &= \frac{\sqrt{rs}}{2\kappa} \left(\frac{\kappa^2}{2rs} J_1 + J_2 \right), \\
J_9 &= J_1. \quad (33)
\end{aligned}$$

Three of the J_i functions, namely J_4 , J_5 , and J_8 , are CP-odd and vanish in the SM at tree level. From the two independent functions among these three, one could determine the CP-odd effective couplings $\hat{\alpha}_{A\tilde{Z}}$ and $\hat{\alpha}_{Z\tilde{Z}}$. From the remaining four CP-even angular functions, one obtains information on the anomalous couplings $\hat{\alpha}_{1,2}^{\text{eff}}$, $\hat{\alpha}_{AZ}$, and $\hat{\alpha}_{ZZ}$. The explicit expressions for the J functions in terms of the effective couplings are collected in App. B. We will use them to get analytic insight into the numerical analysis presented below.

⁴Our expression agrees with Ref. [24] with adjustments for the different definitions of angles and form factors. In particular, we have $\theta_1 \rightarrow \pi - \alpha$ and $\theta_2 \rightarrow \beta$, $H_{1,V} \rightarrow 2F_1 G_V$, $H_{1,A} \rightarrow -2F_1 G_A$, $H_{2,V} \rightarrow H_V$, $H_{2,A} \rightarrow -H_A$, $H_{3,V} \rightarrow -K_V$, and $H_{3,A} \rightarrow K_A$. Note that the definitions of J_1 and J_2 are different.

3.2 Observables

Integrating over the three angles in Eq. (21), the differential decay rate as a function of the di-lepton invariant mass is given by

$$\frac{d\Gamma}{dq^2} = \frac{32\pi}{9} \frac{1}{m_H} \mathcal{N}(q^2) (4J_1 + J_2). \quad (34)$$

This observable has been explored recently in Refs. [21, 23]. Here, instead, the main focus is on the angular asymmetries from which individual J functions can be extracted. Some of these asymmetries have already been discussed in Ref. [24]. We define the following angular asymmetries normalized to $d\Gamma/dq^2$:

$$\begin{aligned} \mathcal{A}_{\theta_1} &= \frac{1}{d\Gamma/dq^2} \int_{-1}^1 d\cos\theta_1 \operatorname{sgn}(\cos(2\theta_1)) \frac{d^2\Gamma}{dq^2 d\cos\theta_1} \\ &= 1 - \frac{5}{2\sqrt{2}} + \frac{3J_1}{\sqrt{2}(4J_1 + J_2)}, \\ \mathcal{A}_{\phi}^{(1)} &= \frac{1}{d\Gamma/dq^2} \int_0^{2\pi} d\phi \operatorname{sgn}(\sin\phi) \frac{d^2\Gamma}{dq^2 d\phi} = \frac{9\pi}{32} \frac{J_4}{4J_1 + J_2}, \\ \mathcal{A}_{\phi}^{(2)} &= \frac{1}{d\Gamma/dq^2} \int_0^{2\pi} d\phi \operatorname{sgn}(\sin(2\phi)) \frac{d^2\Gamma}{dq^2 d\phi} = \frac{2}{\pi} \frac{J_8}{4J_1 + J_2}, \\ \mathcal{A}_{\phi}^{(3)} &= \frac{1}{d\Gamma/dq^2} \int_0^{2\pi} d\phi \operatorname{sgn}(\cos\phi) \frac{d^2\Gamma}{dq^2 d\phi} = \frac{9\pi}{32} \frac{J_6}{4J_1 + J_2}, \\ \mathcal{A}_{\phi}^{(4)} &= \frac{1}{d\Gamma/dq^2} \int_0^{2\pi} d\phi \operatorname{sgn}(\cos(2\phi)) \frac{d^2\Gamma}{dq^2 d\phi} = \frac{2}{\pi} \frac{J_9}{4J_1 + J_2}. \end{aligned} \quad (35)$$

The sign function is $\operatorname{sgn}(\pm|x|) = \pm 1$. We further define the double forward-backward asymmetry

$$\begin{aligned} \mathcal{A}_{c\theta_1, c\theta_2} &= \frac{1}{d\Gamma/dq^2} \int_{-1}^1 d\cos\theta_1 \operatorname{sgn}(\cos\theta_1) \int_{-1}^1 d\cos\theta_2 \operatorname{sgn}(\cos\theta_2) \frac{d^3\Gamma}{dq^2 d\cos\theta_1 d\cos\theta_2} \\ &= \frac{9}{16} \frac{J_3}{4J_1 + J_2}. \end{aligned} \quad (36)$$

The single forward-backward asymmetry in the angle θ_2 (see App. A.1), i.e.,

$$\frac{1}{d\Gamma/dq^2} \int_{-1}^1 d\cos\theta_2 \operatorname{sgn}(\cos(\theta_2)) \frac{d^2\Gamma}{dq^2 d\cos\theta_2}, \quad (37)$$

vanishes for $H \rightarrow Z(\rightarrow \ell^+\ell^-)\ell^+\ell^-$ in the present approximation, as already noticed in Ref. [24]. This is different from the analogous forward-backward asymmetry in the electroweak penguin decay $B \rightarrow K^*\ell^+\ell^-$, despite a very similar form factor structure. To understand this difference, we note the explicit expression for the $B \rightarrow K^*\ell^+\ell^-$ decay amplitude in the factorization approximation (sufficient for the purpose of explanation),

$$\begin{aligned} \mathcal{M}(B \rightarrow K^*\ell^+\ell^-) &\propto \bar{u}(q_2) \left[\gamma^\mu \left(C_9^{\text{eff}} + C_{10} \gamma_5 \right) \right] v(q_1) \langle K^*(p) | \bar{s} \gamma_\mu (1 - \gamma_5) b | B(p+q) \rangle \\ &\propto \bar{u}(q_2) \left[\gamma^\mu \left(C_9^{\text{eff}} + C_{10} \gamma_5 \right) \right] v(q_1) \left\{ \frac{2V(q^2)}{m_B + m_{K^*}} i\varepsilon_{\mu\nu\rho\sigma} \epsilon_{K^*}^\nu p^\rho q^\sigma \right\} \end{aligned}$$

$$+(m_B + m_{K^*}) A_1(q^2) \left[\epsilon_{K^* \mu} - \frac{\epsilon_{K^*} \cdot q}{q^2} q_\mu \right] + \dots \Big\}, \quad (38)$$

where V and A_1 denote $B \rightarrow K^*$ form factors. The forward-backward asymmetry is determined by [50]

$$A_{FB}(B \rightarrow K^* \ell^+ \ell^-) \propto \text{Re} \left(A_{\parallel}^L A_{\perp}^{L*} \right) - (L \rightarrow R), \quad (39)$$

where the transversity amplitudes within the current approximation are given by

$$A_{\perp}^{L,R} \propto (C_9^{\text{eff}} \mp C_{10}) \frac{V(q^2)}{m_B + m_{K^*}}, \quad A_{\parallel}^{L,R} \propto (C_9^{\text{eff}} \mp C_{10}) \frac{A_1(q^2)}{m_B - m_{K^*}}. \quad (40)$$

The single forward-backward asymmetry in the angle θ_2 is generated by the CP-even part of the interference of transversity amplitudes, and is proportional to $\text{Re}(C_9^{\text{eff}} C_{10}^*)$. Comparing Eq. (38) and Eq. (27), and noting the different factors of i in front of the epsilon symbols, we see that the transversity amplitudes $A_{\perp}^{L,R}$ in $H \rightarrow Z \ell^+ \ell^-$ decays are CP-odd at tree level (when $H_{3,V/A}$ is real), hence the interference of $A_{\perp}^{L,R}$ and $A_{\parallel}^{L,R}$ cannot induce a CP-even observable. This implies the vanishing of the single forward-backward asymmetry in $H \rightarrow Z \ell^+ \ell^-$ decay at tree level.⁵

Due to the vanishing of CP-odd Higgs couplings to gauge bosons in the SM, the angular functions J_4 and J_8 and hence the asymmetries $\mathcal{A}_{\phi}^{(1)}$ and $\mathcal{A}_{\phi}^{(2)}$ are generated only by the anomalous couplings from the $d = 6$ operators. In principle, the asymmetries defined in Eqs. (35) and (36) can determine the six anomalous couplings appearing in $HZ\ell\ell$ form factors unambiguously.

3.3 Higgs couplings in angular asymmetries of $H \rightarrow Z \ell^+ \ell^-$

In this section we discuss the impact of anomalous Higgs couplings on the angular asymmetries in Eqs. (35) and (36) and, for comparison, the di-lepton invariant mass distribution $d\Gamma/dq^2$, Eq. (34). Some of these asymmetries and their sensitivity to new physics have already been explored in Ref. [24]. We comment on their results in the remainder. The analysis can be split into the CP-conserving and CP non-conserving parts. At order $\mathcal{O}(1/\Lambda^2)$, the CP-odd couplings, $\hat{\alpha}_{AZ}$ and $\hat{\alpha}_{Z\tilde{Z}}$, contribute only to J_4 , J_5 , and J_8 and therefore do not contribute to the decay rate $d\Gamma/ds$. The CP-even couplings $\hat{\alpha}_{AZ}$, $\hat{\alpha}_{ZZ}$, as well as the combinations $\hat{\alpha}_1^{\text{eff}}$ and $\hat{\alpha}_2^{\text{eff}}$, that contain the contact $HZ\ell\ell$ interactions, enter the remaining angular functions J_1 , J_2 , J_3 , J_6 , J_7 , J_9 .

Most of the distinctive phenomenology of the angular asymmetries stems from the suppression of the vector $Z\ell\ell$ coupling g_V compared to the axial coupling g_A . With the conventions of Eq. (15), $g_V \simeq 0.012$ and $g_A \simeq 15g_V$. Inspecting the explicit expressions of J_1 and J_2 in App. B, one sees that $\hat{\alpha}_{AZ}$ and $\hat{\alpha}_1^{\text{eff}}$ contributions come with suppression factors of g_V and g_V^2 , respectively, and therefore have little effect on $d\Gamma/dq^2 \propto (4J_1 + J_2)$. In contrast, in J_3 and J_6 , the contributions from $\hat{\alpha}_{AZ}$ and $\alpha_{\Phi\ell}^V$ are $1/g_V$ enhanced in comparison with the other $d = 6$ couplings. The asymmetries that probe these coefficient functions, $\mathcal{A}_{c\theta_1, c\theta_2}$ and $\mathcal{A}_{\phi}^{(3)}$, are therefore good candidates to reveal effects

⁵ Beyond the narrow-width approximation, complex form factors and a forward-backward asymmetry can also be generated by the imaginary part of the Z boson propagator [51], at the cost of an additional Γ_Z/m_Z suppression.

that would not be visible in the di-lepton mass spectrum. This pattern motivates our focus on two main scenarios for the CP-even sector below. In the first we allow only for non-vanishing $\hat{\alpha}_{\Phi\ell}^{V,A}$ and in the second we consider non-zero $\hat{\alpha}_{AZ}$. In both cases we set all other anomalous couplings to zero.

The contribution of $\hat{\alpha}_{ZZ}$ to J_3 and J_6 is g_V suppressed compared to $\hat{\alpha}_{AZ}$. We therefore anticipate a small effect in the asymmetries from this coupling. Nevertheless, it can compete with $\hat{\alpha}_{\Phi\ell}^A$ and $\hat{\alpha}_{AZ}$ in $d\Gamma/dq^2$ and in the total cross section $\sigma(s)$ of $e^+e^- \rightarrow HZ$. Finally, the coupling $\hat{\alpha}_{ZZ}^{(1)}$ amounts to a global shift of the SM $H \rightarrow ZZ$ vertex. Its effect on asymmetries is again small since it is not enhanced with respect to the SM contribution by numerical factors or $1/g_V$ terms. Since $\hat{\alpha}_{ZZ}$ and $\hat{\alpha}_{ZZ}^{(1)}$ have essentially no impact on the angular asymmetries we do not consider specific scenarios for them but we will comment on their contributions to $d\Gamma/dq^2$ and $\sigma(s)$.

The $d = 6$ corrections to the couplings g_A and g_V are taken into account and are shown explicitly in expressions in this section. To make a clear distinction we define

$$g_A \equiv \bar{g}_A (1 + \delta g_A), \quad g_V \equiv \bar{g}_V \left(1 - \frac{\bar{g}_A}{\bar{g}_V} \delta g_V \right), \quad (41)$$

where $\bar{g}_{V,A}$ are the following combinations of input parameters (free of $d = 6$ corrections):

$$\bar{g}_A \equiv \frac{m_Z}{2} (\sqrt{2} G_F)^{1/2}, \quad \bar{g}_V \equiv \bar{g}_A (1 - 4s_W^2). \quad (42)$$

Throughout this section the $d = 6$ corrections to the electromagnetic vertex can be neglected because they always appear in combination with the $d = 6$ $HZ\gamma$ coupling, which is already $\mathcal{O}(1/\Lambda^2)$.

In the plots in the remainder of this section we show as a shaded band the region $0 \leq q^2 \leq (12 \text{ GeV})^2$ (or $s \lesssim 0.0091$) where the decay $(Z^*, \gamma^*) \rightarrow \ell^+ \ell^-$ is dominated by intermediate $q\bar{q}$ hadronic resonances and our calculation is not valid.⁶ We refer to Ref. [52] for a discussion of the low- q^2 part of the spectrum.

3.3.1 Contact $HZ\ell\ell$ interactions

First, we concentrate on the observability of the contact $HZ\ell\ell$ interaction, setting the other anomalous couplings to zero. The relevant couplings $\hat{\alpha}_{\Phi\ell}^V$ and $\hat{\alpha}_{\Phi\ell}^A$ are defined in Eq. (14) in terms of the $d = 6$ Lagrangian couplings. They enter the form factors $H_{1,V/A}$, which are non-vanishing already in the SM, through the combinations $\hat{\alpha}_{1,2}^{\text{eff}}$, and implicitly through the $Z\ell\ell$ couplings $g_{V,A}$ according to Eq. (16).

We begin our discussion by focusing on the vector contact interaction, that is, we put $\hat{\alpha}_{\Phi\ell}^A = 0$ for the moment, which in our operator basis amounts to $\hat{\alpha}_{\Phi e} = (\hat{\alpha}_{\Phi\ell}^{(1)} + \hat{\alpha}_{\Phi\ell}^{(3)})$. Due to the g_V suppression, the impact of the vector contact interaction in J_1 and J_2 and hence $d\Gamma/ds$ is small. This is confirmed in Fig. 2(a) which, besides the SM result, shows two (barely visible) curves that describe the modifications for the maximally and minimally allowed values in the range of Eq. (18). To understand this analytically we exploit here and below the hierarchy $g_V \ll g_A$ to write simplified expressions⁷ for the angular functions that exhibit the dominant effects in $d\Gamma/ds$ and the asymmetries. We also employ the approximation $r \approx 1/2$, which is correct up to 5%, and make use of the

⁶In experimental studies this region is removed by means of a kinematic cut on the value of q^2 [4, 29].

⁷In the figures, however, we always use the exact expressions, not the simplified versions.

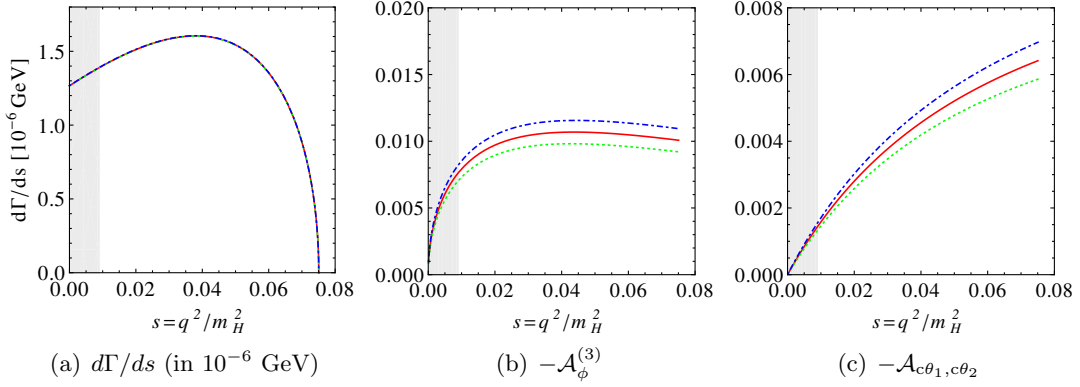


Figure 2: (a) $d\Gamma/ds$, (b) $-\mathcal{A}_\phi^{(3)}$, (c) $-\mathcal{A}_{c\theta_1, c\theta_2}$. Three scenarios are considered. The red solid-line is the SM case. The dotted green line corresponds to $(\hat{\alpha}_{\Phi\ell}^V, \hat{\alpha}_{\Phi\ell}^A) = (-5, 0) \times 10^{-3}$, and the dot-dashed blue line to $(\hat{\alpha}_{\Phi\ell}^V, \hat{\alpha}_{\Phi\ell}^A) = (5, 0) \times 10^{-3}$. The shaded bands exclude $\sqrt{q^2} < 12$ GeV, where hadronic resonances dominate.

fact — appropriate for $H \rightarrow Z\ell^+\ell^-$ — that $s \ll 1$. Within these approximations, the combination $4J_1 + J_2$ that enters $d\Gamma/ds$ in Eq. (34) can be written as

$$4J_1 + J_2 \simeq \sqrt{2} m_H^2 G_F \bar{g}_A^4 (1 + 16s) \times \left[1 + 2\hat{\alpha}_{ZZ}^{(1)} - \frac{48s}{1 + 16s} \hat{\alpha}_{ZZ} + 4 \left(\delta g_A - \frac{\bar{g}_V}{\bar{g}_A} \delta g_V \right) + 2(1 - 2s) \left(\hat{\alpha}_{\Phi\ell}^A - \frac{\bar{g}_V}{\bar{g}_A} \hat{\alpha}_{\Phi\ell}^V \right) \right]. \quad (43)$$

This expression is valid including terms of order $\mathcal{O}(s)$. In the scenario considered here where $\hat{\alpha}_{ZZ} = \hat{\alpha}_{ZZ}^{(1)} = 0$, the corrections to the SM result are very small. This is due to the g_V suppression of the $\alpha_{\Phi\ell}^V$ terms in Eq. (43) (both the direct contribution and the indirect one due to δg_V). In fact, the simplified formula shows that in a generic situation $\hat{\alpha}_{ZZ}, \hat{\alpha}_{ZZ}^{(1)}$ and the axial contact interaction are expected to be more important than the vector contact interaction. However, none of the anomalous couplings is enhanced relative to the SM contribution.

In contrast, the asymmetries $\mathcal{A}_\phi^{(3)}, \mathcal{A}_{c\theta_1, c\theta_2}$ proportional to J_3 and J_6 are sensitive to the vector contact coupling since this and only this contribution is enhanced by $1/g_V$. The results of Fig. 2(b) and 2(c) display the corresponding larger sensitivity to $\alpha_{\Phi\ell}^V$. However, although larger than in $d\Gamma/ds$, the contact interaction is still a small correction of $\mathcal{O}(10)\%$ to the SM result. Larger asymmetries were obtained in Ref. [24], since they allowed larger values of $\hat{\alpha}_{\Phi\ell}^V$. While we formally agree with their results, the estimate Eq. (18) excludes these values of $\hat{\alpha}_{\Phi\ell}^V$.

The above observations can be easily understood from the simplified expressions for the angular functions J_3 and J_6 , which we can write as

$$\begin{aligned} J_3 &\simeq -64\sqrt{2} m_H^2 G_F \bar{g}_A^2 \bar{g}_V^2 s \left(1 - \hat{\alpha}_{\Phi\ell}^A + \frac{\bar{g}_A}{\bar{g}_V} \hat{\alpha}_{\Phi\ell}^V \right), \\ J_6 &\simeq -32 m_H^2 G_F \bar{g}_A^2 \bar{g}_V^2 \sqrt{s} \left(1 - \hat{\alpha}_{\Phi\ell}^A + \frac{\bar{g}_A}{\bar{g}_V} \hat{\alpha}_{\Phi\ell}^V \right). \end{aligned} \quad (44)$$

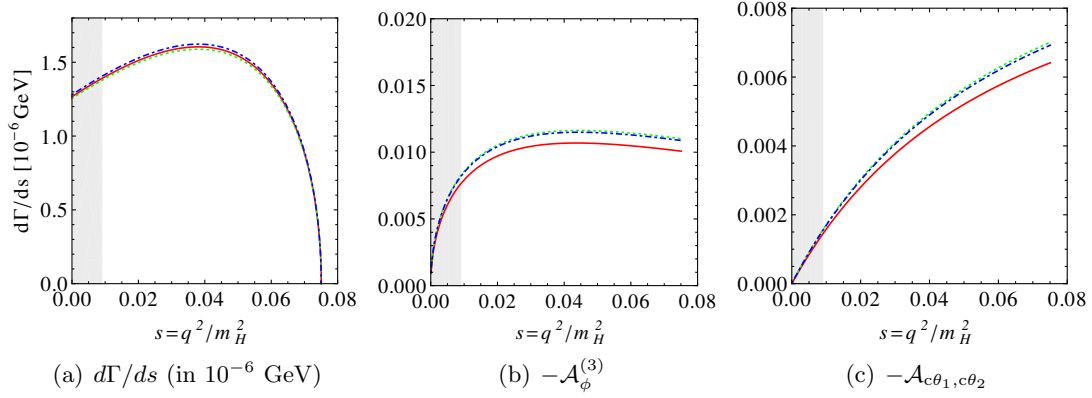


Figure 3: (a) $d\Gamma/ds$, (b) $-\mathcal{A}_\phi^{(3)}$, (c) $-\mathcal{A}_{c\theta_1, c\theta_2}$. The red solid-line is the SM case. The dotted green line corresponds to $(\hat{\alpha}_{\Phi\ell}^V, \hat{\alpha}_{\Phi\ell}^A) = (5, 5) \times 10^{-3}$, whereas the dot-dashed blue line to $(\hat{\alpha}_{\Phi\ell}^V, \hat{\alpha}_{\Phi\ell}^A) = (5, -5) \times 10^{-3}$.

In the last expressions we put $\hat{\alpha}_{ZZ} = \hat{\alpha}_{ZZ}^{(1)} = 0$ and used Eq. (16) to fix $\delta g_{V,A} = -\hat{\alpha}_{\Phi\ell}^{V,A}$. Including the contributions from the denominator in their definition, the asymmetries $\mathcal{A}_\phi^{(3)}$ and $\mathcal{A}_{c\theta_1, c\theta_2}$ are approximated by

$$\begin{aligned} -\mathcal{A}_\phi^{(3)} &\simeq \frac{9\pi\sqrt{2}\bar{g}_V^2}{2\bar{g}_A^2} \frac{\sqrt{s}}{1+16s} \left(1 + \hat{\alpha}_{\Phi\ell}^A + \frac{\bar{g}_A}{\bar{g}_V} \hat{\alpha}_{\Phi\ell}^V \right), \\ -\mathcal{A}_{c\theta_1, c\theta_2} &\simeq \frac{36\bar{g}_V^2}{\bar{g}_A^2} \frac{s}{1+16s} \left(1 + \hat{\alpha}_{\Phi\ell}^A + \frac{\bar{g}_A}{\bar{g}_V} \hat{\alpha}_{\Phi\ell}^V \right). \end{aligned} \quad (45)$$

These asymmetries are largely dominated by the vector contact interaction enhanced by the factor $\bar{g}_A/\bar{g}_V \approx 15$. All other effects are subleading (including those arising from the denominator, given in Eq. (43)). Unfortunately, the asymmetries proportional to these functions are intrinsically small, because they contain a global \bar{g}_V^2 factor. Note that in the denominators of the last expressions the term $16s$ is of order one and cannot be expanded. This term is largely responsible for the shape of the curves in Figs. 2(b) and 2(c). Interestingly, in the absence of other anomalous couplings the ratio between these two asymmetries,

$$\frac{\mathcal{A}_{c\theta_1, c\theta_2}}{\mathcal{A}_\phi^{(3)}} \simeq \frac{4\sqrt{2}s}{\pi}, \quad (46)$$

is given by pure kinematics, independent of $d = 6$ corrections.

We now turn to the more general case where both the vector and axial-vector $HZ\ell\ell$ couplings contribute. Fig. 3 shows $d\Gamma/ds$ and the same asymmetries as Fig. 2 for two values of the axial coupling $\hat{\alpha}_{\Phi\ell}^A$ and fixed $\hat{\alpha}_{\Phi\ell}^V = 0.005$. The essential features can be easily understood with the help of the approximate expressions of Eqs. (43), (44), and (45). From Eq. (43) we see that $\hat{\alpha}_{\Phi\ell}^A$ gives the dominant contribution in the $d = 6$ corrections to $d\Gamma/ds$, but the fact that $\hat{\alpha}_{\Phi\ell}^A$ cannot exceed a few permille makes the modifications to SM result very small (Fig. 3(a)). In the asymmetries, Figs. 3(b) and 3(c), the deviations from the SM are essentially due to $1/g_V$ enhanced contribution from $\hat{\alpha}_{\Phi\ell}^V$. The inclusion of $\hat{\alpha}_{\Phi\ell}^A$ for fixed $\hat{\alpha}_{\Phi\ell}^V$ barely alters this result.

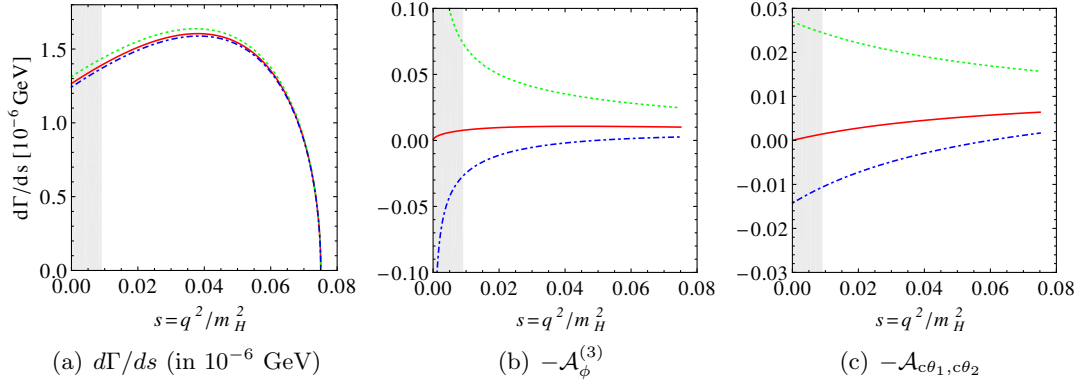


Figure 4: (a) $d\Gamma/ds$, (b) $-\mathcal{A}_\phi^{(3)}$, (c) $-\mathcal{A}_{c\theta_1, c\theta_2}$. Three scenarios are considered. The red solid-line is the SM case. The dot-dashed blue line corresponds to $\hat{\alpha}_{AZ} = -1.3 \times 10^{-2}$, whereas the dotted green line corresponds to $\hat{\alpha}_{AZ} = 2.6 \times 10^{-2}$.

3.3.2 Anomalous $HZ\gamma$ coupling

We next investigate the effect of the anomalous $HZ\gamma$ coupling, $\hat{\alpha}_{AZ}$, and display it for the two values $\hat{\alpha}_{AZ} = -0.013$ and $\hat{\alpha}_{AZ} = 0.026$, which limit the allowed range Eq. (19) as discussed in Sec. 2.

The simplified expression relevant to $d\Gamma/ds$ is given by

$$4J_1 + J_2 \simeq \sqrt{2} m_H^2 G_F \bar{g}_A^4 (1 + 16s) \left(1 - \frac{12 \bar{g}_V g_{\text{em}} Q_\ell}{\bar{g}_A^2 (1 + 16s)} \hat{\alpha}_{AZ} \right), \quad (47)$$

from which we immediately deduce that the overall effect on $d\Gamma/ds$ is very small due to the \bar{g}_V suppression of the $\hat{\alpha}_{AZ}$ term (see Fig. 4(a)). Remarkably however, despite a \bar{g}_V^2 suppression, the asymmetry $\mathcal{A}_\phi^{(3)}$, Fig. 4(b), can reach 5% for values of $\hat{\alpha}_{AZ}$ close to the upper bound. The effect is less pronounced in the asymmetry $\mathcal{A}_{c\theta_1, c\theta_2}$, but can reach the percent level, see Fig. 4(c). The larger effect in the asymmetries is due to the fact that in J_3 and J_6 the $\hat{\alpha}_{AZ}$ contribution is $1/g_V$ enhanced. Moreover, the photon pole in $\mathcal{A}_\phi^{(3)}$ is only partially cancelled by the \sqrt{s} factor. These features become evident from the approximate expressions

$$\begin{aligned} -\mathcal{A}_\phi^{(3)} &\simeq \frac{9\pi\sqrt{2}\bar{g}_V^2}{2\bar{g}_A^2} \frac{\sqrt{s}}{1+16s} \left(1 - \frac{g_{\text{em}} Q_\ell}{8\bar{g}_V s} \hat{\alpha}_{AZ} \right), \\ -\mathcal{A}_{c\theta_1, c\theta_2} &\simeq \frac{36\bar{g}_V^2}{\bar{g}_A^2} \frac{s}{1+16s} \left(1 - \frac{g_{\text{em}} Q_\ell}{4\bar{g}_V s} \hat{\alpha}_{AZ} \right). \end{aligned} \quad (48)$$

The double enhancement by the factor $1/(g_V s) \sim \mathcal{O}(10^3)$ implies that these asymmetries can exceed their SM expectation, even when the anomalous couplings are generated by BSM physics in the multi-TeV range. Note that in the presence of the anomalous $\hat{\alpha}_{AZ}$ coupling the ratio of $\mathcal{A}_\phi^{(3)}$ and $\mathcal{A}_{c\theta_1, c\theta_2}$ is no longer free of $d = 6$ corrections.

In the case of the anomalous α_{AZ} coupling, the square of the form factors $H_{i,V/A}$ contains terms proportional to α_{AZ}^2/Λ^4 , which are enhanced by the photon pole for small s . Being formally of higher order in the $1/\Lambda^2$ expansion, they have been consistently

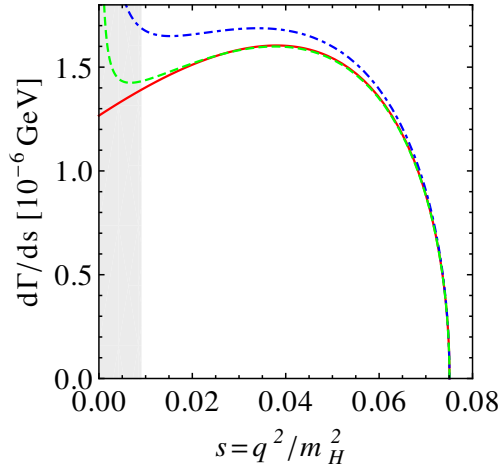


Figure 5: $d\Gamma/ds$ including terms of order $\mathcal{O}(1/\Lambda^4)$ in the squared amplitude. The three scenarios of Fig. 4 are considered: the red solid-line is the SM case, the dot-dashed blue line corresponds to $\hat{\alpha}_{AZ} = -1.3 \times 10^{-2}$, whereas the dotted green line corresponds to $\hat{\alpha}_{AZ} = 2.6 \times 10^{-2}$.

neglected up to this point. However, the $1/\Lambda^4$ photon-pole enhanced terms may give the dominant contribution from anomalous couplings, also when the effective Lagrangian is extended to $d = 8$ operators, because they are enhanced by $1/(s g_V)$ with respect to the terms of order $\mathcal{O}(1/\Lambda^2)$. In the specific case of α_{AZ} , it is therefore mandatory to investigate the photon-pole enhanced $1/\Lambda^4$ terms in the expressions for the J functions. The effect on the di-lepton mass distribution $d\Gamma/ds$ is indeed sizeable as can be seen by comparing Fig. 4(a) to Fig. 5, which includes the enhanced $\mathcal{O}(1/\Lambda^4)$ terms. In the low- s region, $d\Gamma/ds$ can now be enhanced by up to 20% above the SM for the larger value of α_{AZ} . The effects on asymmetries are less relevant and affect chiefly the part inside the shaded region $q^2 \leq (12 \text{ GeV})^2$. Therefore, we refrain from displaying the asymmetries in the presence of $\mathcal{O}(1/\Lambda^4)$ terms.

3.3.3 CP-odd couplings

There are two asymmetries that are sensitive to the CP-odd couplings $\hat{\alpha}_{A\tilde{Z}}$ and $\hat{\alpha}_{Z\tilde{Z}}$, $\mathcal{A}_\phi^{(1)}$ and $\mathcal{A}_\phi^{(2)}$, as defined in Eq. (35). The asymmetry $\mathcal{A}_\phi^{(1)}$, proportional to J_4 , is enhanced by a prefactor $1/\sqrt{s}$ at small s . This asymmetry is largely dominated by the coupling $\hat{\alpha}_{A\tilde{Z}}$ due to a suppression by $g_V s$ of the $\hat{\alpha}_{Z\tilde{Z}}$ term. On the other hand, the asymmetry $\mathcal{A}_\phi^{(2)}$, proportional to J_8 , receives contributions from both $\hat{\alpha}_{A\tilde{Z}}$ and $\hat{\alpha}_{Z\tilde{Z}}$. Although the $\hat{\alpha}_{A\tilde{Z}}$ term is multiplied by g_V , the small s factor in front of $\hat{\alpha}_{Z\tilde{Z}}$ renders both contribution to be of the same order. These features can be seen from the approximate expressions

$$\begin{aligned} \mathcal{A}_\phi^{(1)} &\simeq -\frac{9\pi\sqrt{2}}{16} \frac{\sqrt{1-12s}}{\sqrt{s}(1+16s)} \frac{\bar{g}_V g_{\text{em}} Q_\ell}{\bar{g}_A^2} \hat{\alpha}_{A\tilde{Z}}, \\ \mathcal{A}_\phi^{(2)} &\simeq \frac{16\sqrt{1-12s}}{\pi(1+16s)} \left(s \alpha_{Z\tilde{Z}} + \frac{\bar{g}_V g_{\text{em}} Q_\ell}{4\bar{g}_A^2} \hat{\alpha}_{A\tilde{Z}} \right). \end{aligned} \quad (49)$$

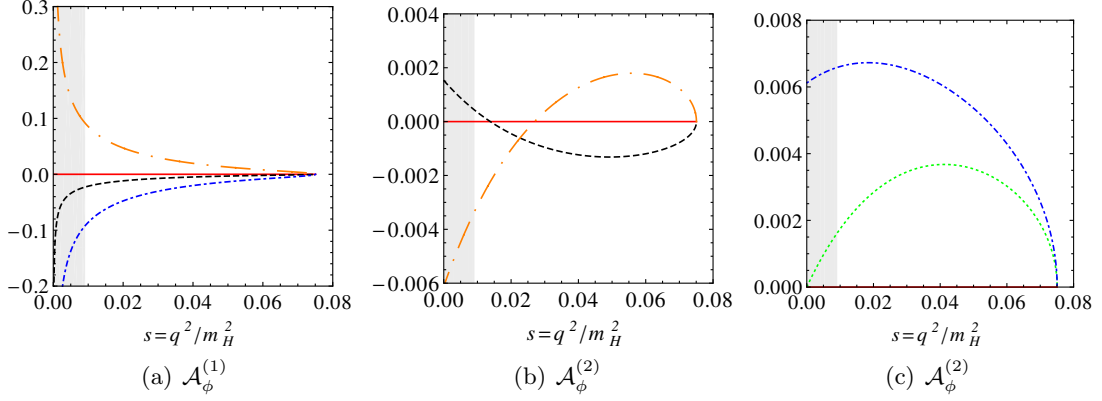


Figure 6: Asymmetries $\mathcal{A}_\phi^{(1,2)}$ in four different scenarios. The dot-long-dashed orange line corresponds to $(\hat{\alpha}_{Z\tilde{Z}}, \hat{\alpha}_{A\tilde{Z}}) = (4, 4) \times 10^{-2}$, the dashed black line to $(\hat{\alpha}_{Z\tilde{Z}}, \hat{\alpha}_{A\tilde{Z}}) = (-2, -1) \times 10^{-2}$, the dot-dashed blue line corresponds to $(\hat{\alpha}_{Z\tilde{Z}}, \hat{\alpha}_{A\tilde{Z}}) = (4, -4) \times 10^{-2}$, and the dotted green line corresponds to $(\hat{\alpha}_{Z\tilde{Z}}, \hat{\alpha}_{A\tilde{Z}}) = (4, 0) \times 10^{-2}$. The solid red line is the vanishing SM result.

The interplay between the two terms can generate an asymmetry-zero in $\mathcal{A}_\phi^{(2)}$, provided both CP-odd couplings have the same sign (recall $Q_\ell = -1$). Its approximate location is at

$$s_0 = -\frac{\bar{g}_V g_{\text{em}} Q_\ell \hat{\alpha}_{A\tilde{Z}}}{4\bar{g}_A^2 \hat{\alpha}_{Z\tilde{Z}}}, \quad (50)$$

the measurement of which would establish a relation between the two CP-odd effective anomalous couplings. We illustrate these results in Fig. 6. For want of stringent experimental bounds on these couplings we assume that they will not exceed a few times 10^{-2} , as is the case for the other couplings previously studied. In Fig 6(a), $\mathcal{A}_\phi^{(1)}$ and three different scenarios for $\hat{\alpha}_{Z\tilde{Z}}$ and $\hat{\alpha}_{A\tilde{Z}}$ are displayed. For lower values of s the asymmetry can be of the order of 10%, but it goes to zero rather quickly at the kinematic end point. This asymmetry is essentially independent of $\hat{\alpha}_{Z\tilde{Z}}$. In Fig. 6(b) and 6(c) we show $\mathcal{A}_\phi^{(2)}$ for four different choices of $\hat{\alpha}_{Z\tilde{Z}}$ and $\hat{\alpha}_{A\tilde{Z}}$. Fig. 6(b) shows two cases where the couplings have same signs and the asymmetry has a zero. The position of the zero can be estimated from the approximate expression Eq. (50). For $\hat{\alpha}_{A\tilde{Z}} = \hat{\alpha}_{Z\tilde{Z}}$ the zero predicted at $s_0 \simeq 0.028$, in good agreement with Fig. 6(b) (dot-long-dashed orange curve). The significance of the asymmetry-zero is somewhat limited in practice, since the asymmetry itself is only at the permille level. The asymmetry, however, could reach a few percent for values of the anomalous couplings one order of magnitude larger, which are not ruled out experimentally [4]. In the next section we show that in $e^+e^- \rightarrow HZ(\rightarrow \ell^+\ell^-)$ the CP-odd couplings can generate the asymmetry $\mathcal{A}_\phi^{(2)}$ at the percent level for anomalous couplings of a few times 10^{-2} .

Similar to the CP-even $HZ\gamma$ coupling, the anomalous coupling $\alpha_{A\tilde{Z}}$ also generates $\mathcal{O}(1/\Lambda^4)$ terms that are photon-pole enhanced. The effect on $d\Gamma/ds$ is similar to the CP-even case shown in Fig. 5 and we do not show the CP-odd case here explicitly. The asymmetries are again less affected by the $1/\Lambda^4$, and change mostly in the shaded region $q^2 \leq (12 \text{ GeV})^2$.

4 Angular asymmetries of $e^+e^- \rightarrow HZ(\rightarrow \ell^+\ell^-)$

From the result for the decay $H \rightarrow Z\ell^+\ell^-$ it is straightforward to calculate the cross section for the crossing-symmetric process $e^+e^- \rightarrow HZ(\rightarrow \ell^+\ell^-)$. In order to fully exploit crossing symmetry we define the kinematics for $e^+e^- \rightarrow HZ(\rightarrow \ell^+\ell^-)$ as discussed in App. A.2. In particular, the angles θ_1 , θ_2 and ϕ are now defined as in Fig. 14. According to these definitions, the cross section can be written in terms of the same function $\mathcal{J}(q^2, \theta_1, \theta_2, \phi)$. The process is described by the same set of form factors $H_{i,V/A}$ and angular functions J_i , see Eq. (31), analytically continued in the energy s to describe the different kinematic regime. The main difference between the two processes is that the di-lepton invariant mass $q^2 = s m_H^2$ is now given by the CM energy of the initial-state e^+e^- pair. The differential cross section for $e^+e^- \rightarrow HZ(\rightarrow \ell^+\ell^-)$ is therefore expressed as before as

$$\frac{d\sigma}{d\cos\theta_1 d\cos\theta_2 d\phi} = \frac{1}{m_H^2} \mathcal{N}_\sigma(q^2) \mathcal{J}(q^2, \theta_1, \theta_2, \phi), \quad (51)$$

where the new normalisation reads

$$\mathcal{N}_\sigma(q^2) = \frac{1}{2^{10}(2\pi)^3} \frac{1}{\sqrt{r} \gamma_Z} \frac{\sqrt{\lambda(1, s, r)}}{s^2}. \quad (52)$$

Note that we still use the Higgs mass to construct the dimensionless variables s , r , and γ_Z , as in Eq. (24).

The threshold energy for the reaction is given by $\sqrt{q_{\text{th}}^2} = (m_H + m_Z) \approx 217$ GeV which gives, in units of m_H^2 , the minimal s value

$$s_{\text{th}} = q_{\text{th}}^2/m_H^2 \approx 2.98. \quad (53)$$

The form factors are therefore probed at much higher energies, which leads to non-trivial phenomenological consequences in comparison with $H \rightarrow Z\ell^+\ell^-$. We limit our numerical analysis to intermediate energies accessible to a first-stage high-energy e^+e^- collider, and study the range

$$s_{\text{th}} \leq s \leq 7.0, \quad (54)$$

which translates into $q_{\text{th}}^2 \leq q^2 \lesssim (332 \text{ GeV})^2$. Depending on the value of the BSM scale Λ , the effective Lagrangian description ceases to be valid for very high values of s . In the theoretical expressions for the production process, this is seen from the fact that the $d=6$ corrections relative to the SM generally contain terms of order $s\alpha_k$. The above chosen range for s guarantees that the EFT description is valid when Λ is above 1 TeV.

The total $e^+e^- \rightarrow HZ$ cross section is given by

$$\sigma(s) = \frac{32\pi}{9} \frac{1}{m_H^2} \mathcal{N}_\sigma(4J_1 + J_2). \quad (55)$$

We define angular asymmetries analogous to those of Eqs. (35) and (36), normalizing them by the total cross section. Since the normalization \mathcal{N}_σ drops out in the ratios, the final expression for the asymmetries in terms of J functions are identical to those of $H \rightarrow Z\ell^+\ell^-$.

The SM cross section can be used to estimate the number of produced events. At $\sqrt{q^2} = 250$ GeV and with an integrated luminosity of 250 fb^{-1} one expects around

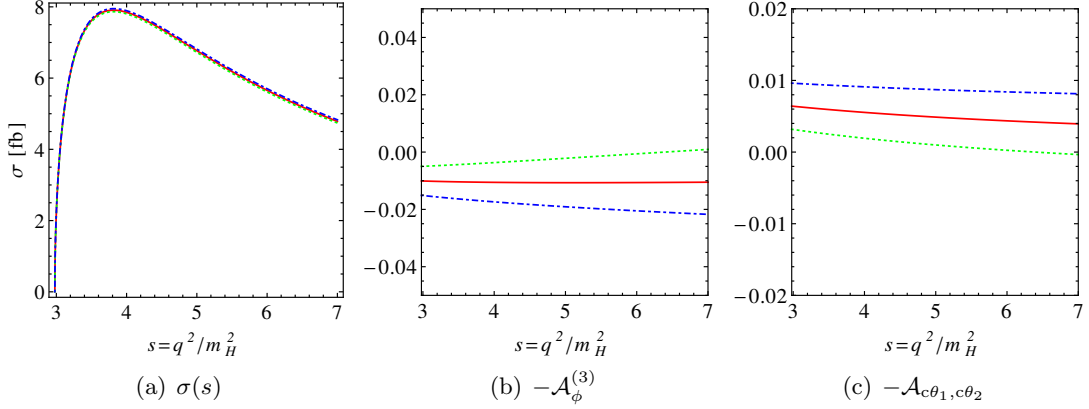


Figure 7: (a) $\sigma(s)$, (b) $-\mathcal{A}_\phi^{(3)}$, (c) $-\mathcal{A}_{c\theta_1, c\theta_2}$. Three scenarios are considered. The red solid-line is the SM case. The dotted green line corresponds to $(\hat{\alpha}_{\Phi\ell}^V, \hat{\alpha}_{\Phi\ell}^A) = (-5, 0) \times 10^{-3}$, and the dot-dashed blue line to $(\hat{\alpha}_{\Phi\ell}^V, \hat{\alpha}_{\Phi\ell}^A) = (5, 0) \times 10^{-3}$.

2300 events, of which up to 1900 could be reconstructed [32] (assuming $H \rightarrow b\bar{b}$). This number decreases to around 1400 for $\sqrt{q^2} = 350$ GeV and integrated luminosity of 350 fb^{-1} due to a decrease in the cross section (see Fig. 7(a)).

In the remainder of this section we study the total cross section and asymmetries to assess their sensitivity to $d = 6$ effective Higgs couplings in analogy with the decay $H \rightarrow Z\ell^+\ell^-$. For the purpose of comparison we consider the same scenarios as in the previous section. Note that in e^+e^- collisions, due to the clean environment, one could also consider Z decay to quarks. The vector and axial-vector couplings should then be replaced by the appropriate values. A detailed anomalous coupling analysis of this possibility is, however, beyond the scope of this paper.

4.1 Contact $HZ\ell\ell$ interactions

We again begin with the case where the axial-vector $HZ\ell\ell$ interactions are set to zero. In Fig. 7 we show results for the same observables and coupling parameter choices that we investigated for $H \rightarrow Z\ell^+\ell^-$. In the total cross section $\alpha_{\Phi\ell}^V$ effects remain g_V suppressed and therefore insignificant, as shown in Fig. 7(a). In the asymmetries, the modification of the SM value due to $\hat{\alpha}_{\Phi\ell}^V$ is more pronounced in $e^+e^- \rightarrow HZ(\rightarrow \ell^+\ell^-)$ than in the decay $H \rightarrow Z\ell^+\ell^-$ due to higher values of s , but the effect is still not dramatic, as shown in Figs. 7(b). The asymmetries can be at most at the level of 1 to 2%.

The situation is more interesting, and different from Higgs decay, when the axial-vector contact interaction is also present. Fig 8(a) shows that the total cross section is quite sensitive to the axial-vector contact coupling. This can be understood with the help of the approximate expression for the combination $4J_1 + J_2$. As before, we exploit $g_V \ll g_A$ and approximate $r = 1/2$, but we can no longer use that s is small. We then find

$$4J_1 + J_2 \simeq \sqrt{2} m_H^2 G_F \bar{g}_A^4 \frac{s+3}{s-1} \times \left[1 + 2\hat{\alpha}_{ZZ}^{(1)} + \frac{12(2s-1)}{s+3} \hat{\alpha}_{ZZ} \right]$$

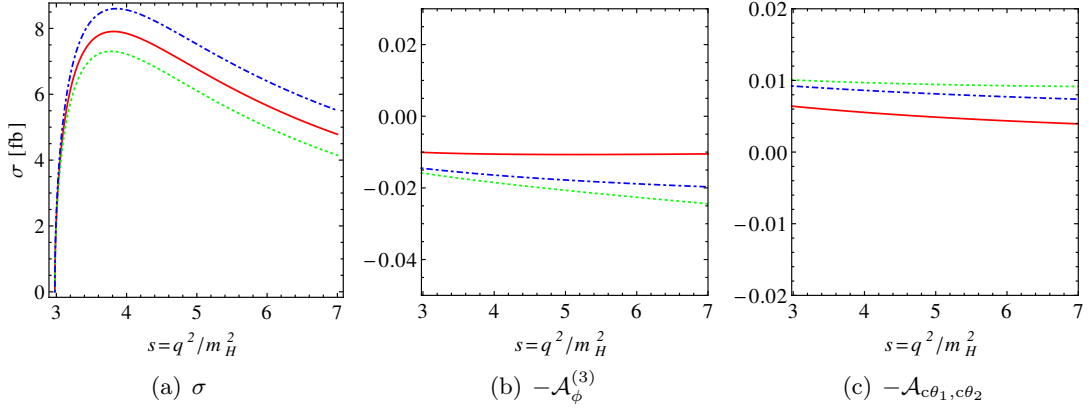


Figure 8: (a) $\sigma(s)$, (b) $-\mathcal{A}_\phi^{(3)}$, (c) $-\mathcal{A}_{c\theta_1, c\theta_2}$. Four scenarios with the same $\hat{\alpha}_{\Phi\ell}^V$ coupling are considered. The red solid-line is the SM case. The dotted green line corresponds to $(\hat{\alpha}_{\Phi\ell}^V, \hat{\alpha}_{\Phi\ell}^A) = (5, 5) \times 10^{-3}$, whereas the dot-dashed blue line to $(\hat{\alpha}_{\Phi\ell}^V, \hat{\alpha}_{\Phi\ell}^A) = (5, -5) \times 10^{-3}$.

$$\begin{aligned}
& -2(2s-1) \left(\hat{\alpha}_{\Phi\ell}^A - \frac{\bar{g}_V}{\bar{g}_A} \hat{\alpha}_{\Phi\ell}^V \right) + 4 \left(\delta g_A - \frac{\bar{g}_V}{\bar{g}_A} \delta g_V \right) \Big] \\
& \simeq \sqrt{2} m_H^2 G_F \bar{g}_A^4 \frac{s+3}{s-1} \left[1 - 2(1+2s) \hat{\alpha}_{\Phi\ell}^A \right]. \tag{56}
\end{aligned}$$

In the last equation we neglected the contributions from $\hat{\alpha}_{\Phi\ell}^V$ that are suppressed by \bar{g}_V and we used that in the adopted scenario $\hat{\alpha}_{ZZ}^{(1)} = \hat{\alpha}_{ZZ} = 0$. Since now $4s \sim \mathcal{O}(10)$, the contribution from $\hat{\alpha}_{\Phi\ell}^A$ is significantly larger than in the invariant mass distribution $d\Gamma/ds$ of $H \rightarrow Z\ell^+\ell^-$. For $\hat{\alpha}_{\Phi\ell}^A = 5 \times 10^{-3}$ the modification of the SM cross section reaches 15% as shown in Fig. 8(a).

The anomalous contributions to the asymmetries $\mathcal{A}_\phi^{(3)}$, $\mathcal{A}_{c\theta_1, c\theta_2}$ shown in Figs. 8(b) and 8(c) are still largely determined by $1/\bar{g}_V$ enhanced $\hat{\alpha}_{\Phi\ell}^V$ contributions. The main dependence on $\hat{\alpha}_{\Phi\ell}^A$ comes from the denominator of the asymmetries, but is subleading compared to the $\hat{\alpha}_{\Phi\ell}^V$ terms from the numerator. For non-vanishing contact couplings the asymmetries are well approximated by

$$\begin{aligned}
-\mathcal{A}_\phi^{(3)} & \simeq -\frac{9\pi\sqrt{2}}{2} \frac{\bar{g}_V^2}{\bar{g}_A^2} \frac{s-1}{2s-1} \frac{\sqrt{s}}{s+3} \left[1 - (1+2s) \left(\hat{\alpha}_{\Phi\ell}^A - \frac{\bar{g}_A}{\bar{g}_V} \hat{\alpha}_{\Phi\ell}^V \right) \right], \\
-\mathcal{A}_{c\theta_1, c\theta_2} & \simeq 9 \frac{\bar{g}_V^2}{\bar{g}_A^2} \frac{1}{s+3} \left[1 - (1+2s) \left(\hat{\alpha}_{\Phi\ell}^A - \frac{\bar{g}_A}{\bar{g}_V} \hat{\alpha}_{\Phi\ell}^V \right) \right]. \tag{57}
\end{aligned}$$

The asymmetries can reach 2% for allowed values of $\hat{\alpha}_{\Phi\ell}^{V,A}$. Relative to the SM value of the asymmetry, the correction from anomalous couplings can still be 100%. The ratio of the asymmetries is determined by kinematics as for $H \rightarrow Z\ell^+\ell^-$.

Thus we conclude that the total cross section $\sigma(s)$ can be significantly modified by $\hat{\alpha}_{\Phi\ell}^A$ but is insensitive to $\hat{\alpha}_{\Phi\ell}^V$ in comparison, while the situation is opposite for (some of) the angular asymmetries. Eq. (56) shows that the cross section of $e^+e^- \rightarrow HZ$ is also quite sensitive to $\hat{\alpha}_{ZZ}$ due to the factor $12(2s-1)/(s+3) \sim \mathcal{O}(20)$. Overall,

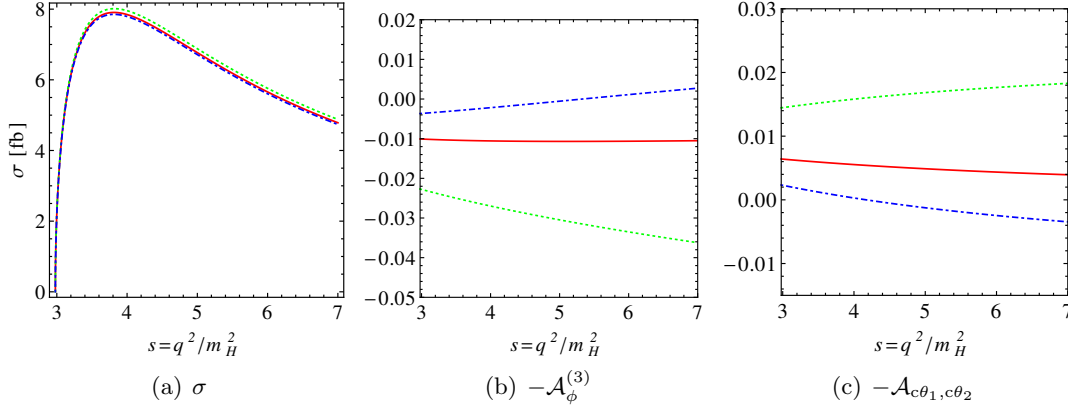


Figure 9: (a) $\sigma(s)$, (b) $-\mathcal{A}_\phi^{(3)}$, (c) $-\mathcal{A}_{c\theta_1, c\theta_2}$. Three scenarios are considered: The red solid-line is the SM result. The dot-dashed blue line corresponds to $\hat{\alpha}_{AZ} = -1.3 \times 10^{-2}$, the dotted green line to $\hat{\alpha}_{AZ} = 2.6 \times 10^{-2}$.

$e^+e^- \rightarrow HZ$ therefore seems to be better suited to discover contact interactions than $H \rightarrow Z\ell^+\ell^-$.

4.2 Anomalous $HZ\gamma$ coupling

Turning to the anomalous $HZ\gamma$ coupling, we find the approximate expression

$$4J_1 + J_2 \simeq \sqrt{2}m_H^2 G_F \bar{g}_A^4 \frac{s+3}{s-1} \left(1 - \frac{s-1}{s+3} \frac{12\bar{g}_V g_{\text{em}} Q_\ell}{\bar{g}_A^2} \hat{\alpha}_{AZ} \right) \quad (58)$$

for the combination of angular functions that determines the cross section. Similar to the case of $H \rightarrow Z\ell^+\ell^-$ the correction is g_V suppressed and has little influence on $\sigma(s)$ as shown in Fig. 9(a).

The asymmetries can reach a few percent (Figs. 9(b) and 9(c)) for the largest allowed values of the $HZ\gamma$ coupling. This again is due to the $1/g_V$ enhancement of the correction. Assuming other couplings to vanish, approximate expressions for the asymmetries in the presence of the $\hat{\alpha}_{AZ}$ coupling are

$$\begin{aligned} -\mathcal{A}_\phi^{(3)} &\simeq -\frac{9\pi\sqrt{2}}{2} \frac{\bar{g}_V^2}{\bar{g}_A^2} \frac{s-1}{2s-1} \frac{\sqrt{s}}{s+3} \left[1 - \frac{g_{\text{em}} Q_\ell (s+1)}{2\bar{g}_V} \hat{\alpha}_{AZ} \right], \\ -\mathcal{A}_{c\theta_1, c\theta_2} &\simeq 9 \frac{\bar{g}_V^2}{\bar{g}_A^2} \frac{1}{s+3} \left[1 - \frac{g_{\text{em}} Q_\ell (s-1)}{\bar{g}_V} \hat{\alpha}_{AZ} \right]. \end{aligned} \quad (59)$$

There is no photon-pole enhancement in this case. Nevertheless, relative to the SM value of the asymmetry, the correction from the anomalous coupling can still be 100%.

4.3 CP-odd couplings

In the $e^+e^- \rightarrow HZ(\rightarrow \ell^+\ell^-)$ case, the asymmetry $\mathcal{A}_\phi^{(1)}$ is again dominated by $\hat{\alpha}_{A\tilde{Z}}$, but the contribution from $\hat{\alpha}_{Z\tilde{Z}}$ is less suppressed than in $H \rightarrow Z\ell^+\ell^-$ due to the larger

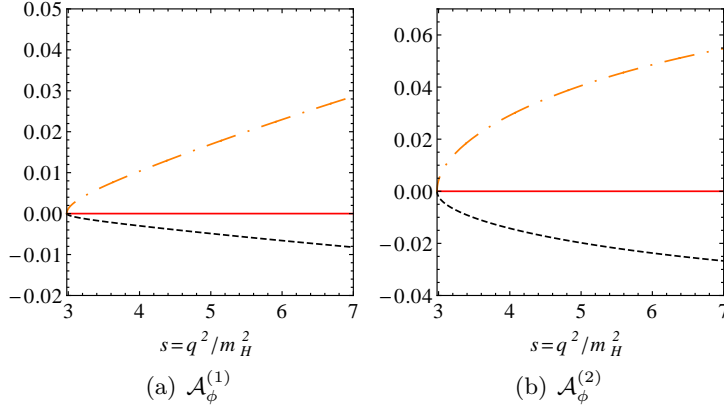


Figure 10: Asymmetries $\mathcal{A}_\phi^{(1,2)}$ in two different scenarios. The dot-dashed orange line corresponds to $(\hat{\alpha}_{Z\tilde{Z}}, \hat{\alpha}_{A\tilde{Z}}) = (4, 4) \times 10^{-2}$, and the dashed black line to $(\hat{\alpha}_{Z\tilde{Z}}, \hat{\alpha}_{A\tilde{Z}}) = (-2, -1) \times 10^{-2}$. The red solid line is the vanishing SM result.

values of s . The situation is opposite for $\mathcal{A}_\phi^{(2)}$, where $\hat{\alpha}_{Z\tilde{Z}}$ dominates and $\hat{\alpha}_{A\tilde{Z}}$ gives a small g_V suppressed contribution. Although a zero may appear in both asymmetries due to the interplay of the two CP-odd couplings, whenever a zero occurs the strong cancellation between the two contributions keeps the asymmetry below the permille level. We therefore provide approximate expressions that contain only the dominant effects:

$$\begin{aligned}\mathcal{A}_\phi^{(1)} &\simeq -\frac{9\pi\sqrt{2}}{8} \frac{\bar{g}_V g_{\text{em}} Q_\ell}{\bar{g}_A^2} \frac{\sqrt{\lambda}(s-1)}{\sqrt{s}(s+3)} \hat{\alpha}_{A\tilde{Z}}, \\ \mathcal{A}_\phi^{(2)} &\simeq \frac{8\sqrt{\lambda}}{\pi(s+3)} \hat{\alpha}_{Z\tilde{Z}}.\end{aligned}\quad (60)$$

From these expressions one sees that the asymmetries can be at the percent level for CP-odd couplings $\mathcal{O}(10^{-2})$. The exact results for the asymmetries $\mathcal{A}_\phi^{(1,2)}$ are shown in Fig. 10 for two coupling value sets.

5 Estimate of SM loop effects

Electroweak one-loop contributions to the processes studied here can be of similar size as the tree-level $d = 6$ corrections discussed in the previous sections. For example, they are around 2% percent for the $H \rightarrow 4\ell$ decay rate [49]. In this section we perform a rough estimate of SM loop contributions and compare them to the effect from the anomalous $HZ\gamma$ coupling. A full analysis is beyond the scope of the present work.

Let us consider the SM one-loop $HZ\gamma^*$ and HZZ^* amplitudes, whose explicit analytical expressions can be found in Refs. [46–48]. The amplitude for the transition $H \rightarrow ZV$ (with $V = Z^*, \gamma^*$) involves five form factors in general. However, when the particles are on-shell or coupled to conserved currents, which is the case of interest here, only two form factors contribute. We therefore write the amplitudes in the form

$$\mathcal{M}_{HZV}^{\mu\nu}(H \rightarrow Z(p)V(q)) = 2m_Z^2 (\sqrt{2}G_F)^{1/2} \left[\frac{q^\mu p^\nu}{m_H^2} D_V(q^2) + g^{\mu\nu} E_V(q^2) \right], \quad (61)$$

where the loop functions $D_{Z,\gamma}$ and $E_{Z,\gamma}$ are functions of q^2 . The tree-level HZZ vertex is treated separately and already included in Eq. (12).⁸

In the previous sections we discussed the modifications of the form factors $H_{1,V}$ and $H_{2,V}$ due to the anomalous $HZ\gamma$ coupling $\hat{\alpha}_{AZ}$. Including the one-loop $H \rightarrow ZV$ amplitudes of Eq. (61) into the defining expression Eq. (27), we find

$$\begin{aligned} H_{1,V} &= \frac{2m_H(\sqrt{2}G_F)^{1/2}r}{s-r}g_V \left[1 + E_Z(q^2) + \frac{Q_\ell g_{\text{em}} \kappa (s-r)}{2rs g_V} \left(\hat{\alpha}_{AZ} - \frac{2r}{\kappa} E_\gamma(q^2) \right) \right], \\ H_{2,V} &= \frac{2m_H(\sqrt{2}G_F)^{1/2}}{s-r}g_V \left[r D_Z(q^2) - \frac{Q_\ell g_{\text{em}}(s-r)}{s g_V} (\hat{\alpha}_{AZ} + r D_\gamma(q^2)) \right], \end{aligned} \quad (62)$$

which should be compared to Eq. (28). For the present purpose we have kept only the anomalous $HZ\gamma$ interaction, setting all other $d=6$ couplings to zero. The terms with an intermediate photon are $1/g_V$ enhanced with respect to the terms with an intermediate Z . Since the one-loop $H \rightarrow ZZ$ amplitude is of the same order as the $H \rightarrow Z\gamma$ amplitude, we can neglect the contributions from D_Z and E_Z in the further discussion.

We start by discussing the modifications to $H_{1,V}$ and $H_{2,V}$ in the decay $H \rightarrow Z\ell^+\ell^-$. The first important observation is that, since the Higgs boson cannot decay into WW or $t\bar{t}$, the loop contribution is real and does not generate an imaginary part of the form factors. Therefore, the angular structures in the presence of these loop contributions remain the same as discussed in the previous sections. Second, the s dependence of the loop contribution is small, since $s \ll 1$. Therefore, the inclusion of the HZV amplitude at one loop amounts, essentially, to shifting the value of $\hat{\alpha}_{AZ}$ by an amount given by the expressions in round brackets in Eq. (62). To estimate the size of this shift in $H_{1,V}$ and $H_{2,V}$, we compare the allowed range for the anomalous $HZ\gamma$ coupling,

$$\hat{\alpha}_{AZ} \in [-1.3, 2.6] \times 10^{-2}, \quad (63)$$

to the quantities

$$\frac{2r}{\kappa} E_\gamma(s=0.01) = -7.1 \times 10^{-3}, \quad (64)$$

and

$$r D_\gamma(s=0.01) = 7.1 \times 10^{-3}, \quad (65)$$

respectively. (The energy dependence of this function is small in the s range relevant to $H \rightarrow Z\ell^+\ell^-$.) The shift is therefore small relative to the allowed values of $\hat{\alpha}_{AZ}$. This is shown explicitly in Fig. 11(a) for

$$\delta H_{1,V} = \frac{Q_\ell g_{\text{em}} \kappa (s-r)}{2rs g_V} \left(\hat{\alpha}_{AZ} - \frac{2r}{\kappa} E_\gamma(q^2) \right), \quad (66)$$

⁸The $d=6$ corrections from the redefinition of the Lagrangian input parameters in the last equation can be neglected since they generate terms that are loop and $1/\Lambda^2$ suppressed. It also needs to be pointed out that the form factors $D_{Z,\gamma}$ and $E_{Z,\gamma}$ are gauge invariant only if both external states are on their mass shells. We use the expressions from Refs. [46–48] where the 't Hooft-Feynman gauge is adopted, and drop the (presumably small) box-diagram contributions to the $H \rightarrow Z\ell^+\ell^-$ and $e^+e^- \rightarrow HZ$ processes, which would be required to restore gauge invariance. In the q^2 range relevant to the decay $H \rightarrow Z\ell^+\ell^-$ the gauge dependence of the one-loop expressions for the $HZ\gamma^*$ amplitude is expected to be small because the photon is nearly on the mass shell in relation to m_H^2 .

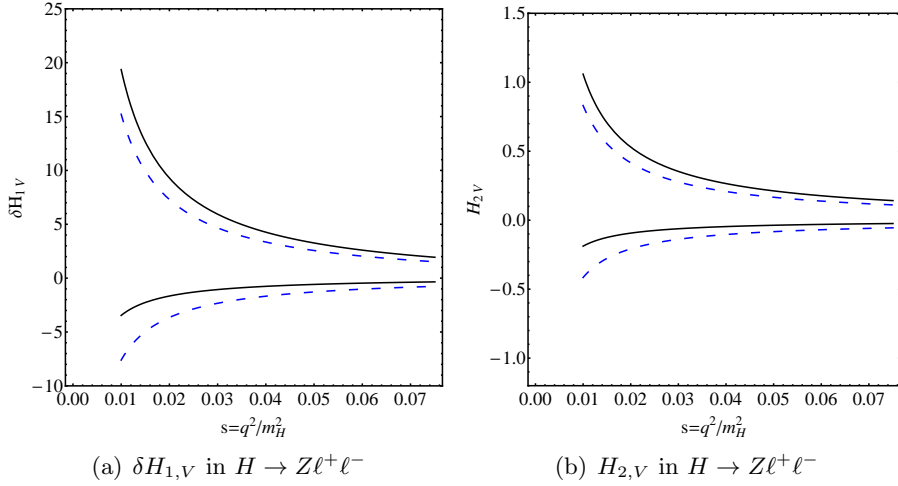


Figure 11: Dominant effects due to the $H \rightarrow ZV$ one loop amplitude in the form factors $H_{1,V}$ and $H_{2,V}$ in decays $H \rightarrow Z\ell^+\ell^-$. In (a) we show $\delta H_{1,V}$, Eq. (66), and (b) $H_{2,V}$. The results within the solid lines include the dominant loop contribution and $\hat{\alpha}_{AZ} \in [-1.3, 2.6] \times 10^{-2}$. Results within the dashed line include solely the effects of $\hat{\alpha}_{AZ}$.

and in Fig. 11(b) for $H_{2,V}$. We therefore conclude that the previously discussed asymmetries are not affected dramatically by loop effects, at least in the study of the anomalous $HZ\gamma$ interaction. In any case, SM loop effects are calculable and should simply be included in a definitive analysis, when sufficient experimental data are available.

Turning to $e^+e^- \rightarrow HZ$, we note that the form factors are now probed in the kinematic range, where the off-shell momentum $q^2 \geq (m_H + m_Z)^2$. The loop functions D_V and E_V develop imaginary parts and therefore the form factors $H_{1,V}$ and $H_{2,V}$ are complex, which generates additional angular structures in Eq. (31). However, while the imaginary parts are sizable, as shown in the two right panels Fig. 12(b) and Fig. 12(d), the real parts of the form factors $H_{1,V}$ and $H_{2,V}$ relevant to the asymmetries discussed in the previous sections, are not dramatically altered and even smaller than for $H \rightarrow Z\ell^+\ell^-$. The corresponding results for $\delta H_{1,V}$ and $H_{2,V}$ are displayed in the two left panels of Fig. 12. Numerically, the contribution from $\frac{2r}{\kappa}E_\gamma(q^2)$ to $\delta H_{1,V}$ now ranges from $(-0.66 - i12) \times 10^{-3}$ near threshold ($s = 3$) to $(1.7 - i5.9) \times 10^{-3}$ at $s = 7$. Similarly $rD_\gamma(q^2)$, which affects $H_{2,V}$ varies from $(1.4 + i11) \times 10^{-3}$ at $s = 3$ to $(-1.5 + i5.8) \times 10^{-3}$ at $s = 7$. The real part of these numbers should again be compared to the range given in Eq. (63).

6 Summary

In this work we studied the observability of anomalous $d = 6$ Higgs couplings in $H \rightarrow Z(\rightarrow \ell^+\ell^-)\ell^+\ell^-$ decay and in the crossing-symmetric process $e^+e^- \rightarrow HZ(\rightarrow \ell^+\ell^-)$. We computed the differential decay width $d\Gamma/dq^2$ of $H \rightarrow Z\ell^+\ell^-$, the total cross section of $e^+e^- \rightarrow HZ(\rightarrow \ell^+\ell^-)$, $\sigma(s)$, as well as angular asymmetries in both processes. Our particular interest regarded the question, see also Ref. [24], whether angular asymmetries

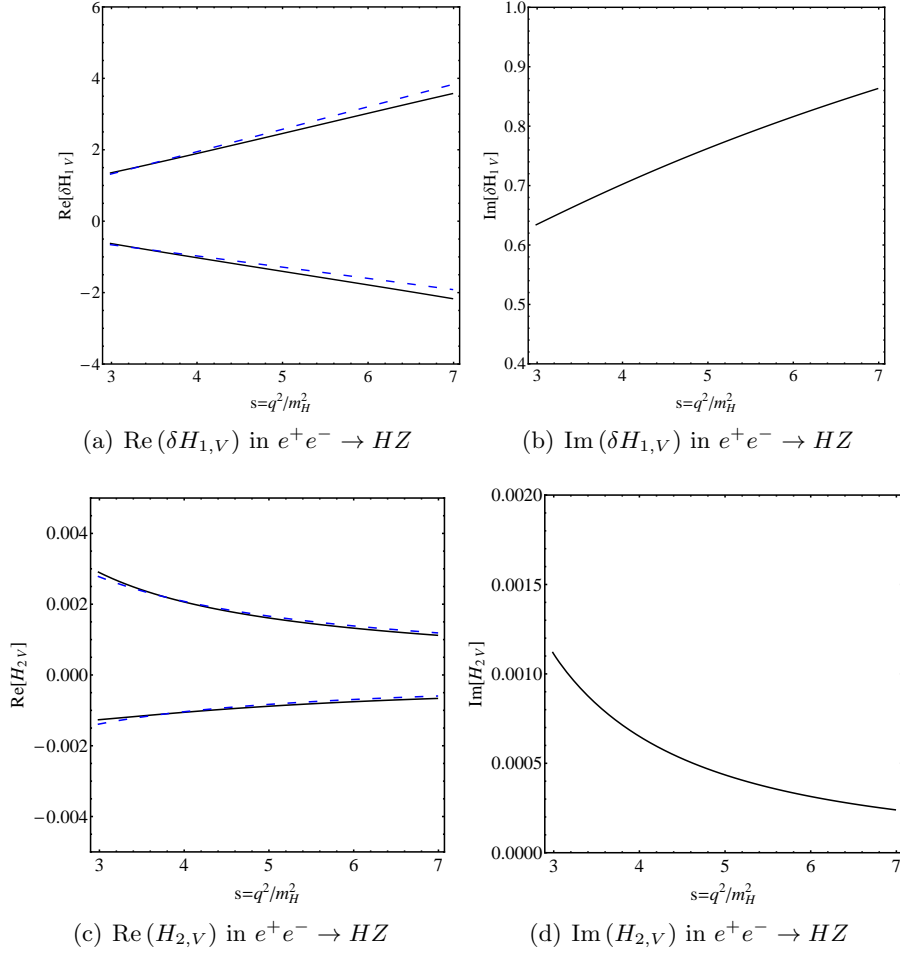


Figure 12: Dominant effects due to the $H \rightarrow ZV$ one loop amplitude in the form factors $H_{1,V}$ and $H_{2,V}$ in $e^+e^- \rightarrow HZ(\rightarrow \ell^+\ell^-)$. In (a) and (b) we show $\text{Re}(\delta H_{1,V})$ and $\text{Im}(\delta H_{1,V})$, respectively. In (c) and (d) we show $\text{Re}(H_{2,V})$ and $\text{Im}(H_{2,V})$ respectively. Results within the solid lines include the dominant loop contribution and $\hat{\alpha}_{AZ} \in [-1.3, 2.6] \times 10^{-2}$. Results within the dashed lines include solely the effects of $\hat{\alpha}_{AZ}$.

have the potential to reveal BSM physics that would be hidden in $d\Gamma/dq^2$ and $\sigma(s)$. In some of these asymmetries, the anomalous $HZ\gamma$ coupling, $\hat{\alpha}_{AZ}$, and the vector contact $HZ\ell\ell$ interaction parametrized by $\hat{\alpha}_{\Phi\ell}^V$, are enhanced with respect to the SM contribution by a factor of $1/g_V$. These two types of interactions are therefore the prime targets of the asymmetry analysis. Our main conclusions can be summarized as follows:

- We identify several angular asymmetries, which are indeed very sensitive to anomalous couplings.
- Within the presently allowed range of the anomalous $HZ\gamma$ interaction strength, $\hat{\alpha}_{AZ}$, modifications of angular asymmetries of $\mathcal{O}(1)$ and even larger relative to the SM value are still possible indicating sensitivity to multi-TeV scales.
- Anomalous $HZ\ell\ell$ contact interactions have smaller effects. This is mainly because

we find that their size is already tightly constrained by existing data, in agreement with the constraints derived in Ref. [38] (although this refers to another operators basis). The effects of the contact $HZ\ell\ell$ interactions in the angular asymmetries of $H \rightarrow Z\ell^+\ell^-$ were previously investigated in Ref. [24]. While we formally agree with their results, we find significantly smaller asymmetries, since the typical values of $\hat{\alpha}_{\Phi\ell}^V$ adopted in that paper are about a factor of four larger than those allowed in the present analysis.

- At present, the CP-odd $d = 6$ couplings are not strongly constrained by data. We showed that CP-odd asymmetry $\mathcal{A}_\phi^{(1)}$ can reach the few percent level in both in $H \rightarrow Z\ell^+\ell^-$ decay and $e^+e^- \rightarrow HZ$ Higgs production. In $H \rightarrow Z\ell^+\ell^-$ an asymmetry-zero may occur. However, for allowed values of the CP-odd couplings the asymmetry that can display this zero is never large.
- Most interesting asymmetries are small in absolute terms, reaching at most 10%, and often much less, because they are suppressed by the small vector $Z\ell\ell$ coupling.
- Overall, the process $e^+e^- \rightarrow HZ$ seems better suited than $H \rightarrow Z\ell^+\ell^-$ for the study of anomalous $HZ\ell\ell$ contact interactions due to the higher di-lepton invariant masses. This is particularly true for the contributions of $\hat{\alpha}_{\Phi\ell}^A$ (as well as of $\hat{\alpha}_{ZZ}$) to the total cross section, where 15% percent modifications are possible. On the other hand, $H \rightarrow Z\ell^+\ell^-$ provides better sensitivity to the anomalous $HZ\gamma$ coupling due to the photon-pole enhancement.

We further provided a rough estimate of SM loop contributions to the processes discussed here. These loop contributions have been calculated in the past and our estimate suggests that loop effects are small compared to the presently allowed $d = 6$ effects. Once sufficient data is available to attempt constraining $d = 6$ couplings from angular asymmetries, SM loop effects should simply be included. However, the experimental detection of angular asymmetries will be challenging even with the planned higher statistics up-grades of the LHC.

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A Kinematics

A.1 $H \rightarrow Z\ell^+\ell^-$

Here we describe the kinematics and the angle conventions used in our results. The reaction is labelled as $H(p_H) \rightarrow \ell_1^-(p_3)\ell_1^+(p_4)Z(p)(\rightarrow \ell_2^-(p_1)\ell_2^+(p_2))$, where we labelled

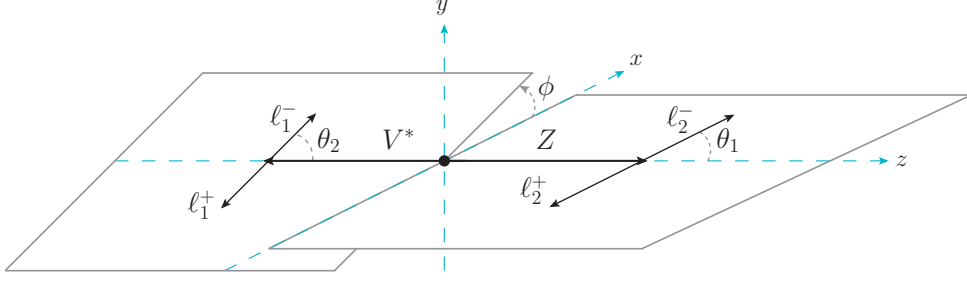


Figure 13: Kinematics of the four-body decay $H \rightarrow Z(\rightarrow \ell^+\ell^-)\ell^+\ell^-$.

the two pairs of leptons to distinguish the pair $\ell_2^-\ell_2^+$ that arises from the decay of the on-shell Z boson from the other. We have

$$p = p_1 + p_2, \quad q = p_3 + p_4, \quad (67)$$

and $p^2 = m_Z^2$. We denote momenta in the $\ell_2^+\ell_2^-$ rest frame by an upper bar (\bar{p}), whereas momenta in the $\ell_1^+\ell_1^-$ rest frame are denoted by an asterisk (p^*).

Using the conventions for the axes given in Fig. 13, we define the positive z direction to be that of the on-shell Z three-momentum \mathbf{p} in the Higgs rest frame. The angle θ_1 is the angle between the momentum \mathbf{p}_1 of ℓ_2^- and the z axis, in the $\ell_2^+\ell_2^-$ rest frame. Accordingly, in the massless limit the momenta $p_{1,2}$ are written in the Z rest frame as

$$\bar{p}_1 = \frac{m_Z}{2}(1, \sin \theta_1, 0, \cos \theta_1), \quad (68)$$

$$\bar{p}_2 = \frac{m_Z}{2}(1, -\sin \theta_1, 0, -\cos \theta_1). \quad (69)$$

The angle θ_2 is the angle between the momentum \mathbf{p}_3 of ℓ_1^- in the $\ell_1^+\ell_1^-$ rest frame and the z axis. The momenta $p_{3,4}$ in the rest frame of the lepton pair are written as

$$p_3^* = \frac{\sqrt{q^2}}{2}(1, \sin \theta_2 \cos \phi, \sin \theta_2 \sin \phi, \cos \theta_2), \quad (70)$$

$$p_4^* = \frac{\sqrt{q^2}}{2}(1, -\sin \theta_2 \cos \phi, -\sin \theta_2 \sin \phi, -\cos \theta_2), \quad (71)$$

where ϕ is the angle between the normal of the planes defined by the z direction and the momenta p_1 and p_3 . It is measured positively from the $\ell_2^+\ell_2^-$ plane to the $\ell_1^+\ell_1^-$ plane.

A.2 $e^+e^- \rightarrow HZ(\rightarrow \ell^+\ell^-)$

The momenta are labelled as $e^-(p_-)e^+(p_+) \rightarrow H(p_H)Z(p)(\rightarrow \ell^-(p_1)\ell^+(p_2))$, where in the final state we kept the conventions used in the $H \rightarrow Z\ell^+\ell^-$. We choose the z direction to be defined by the momentum of the on-shell Z boson in the initial state rest frame, here the incoming e^+e^- rest frame. The xz plane coincides with the plane defined by \mathbf{p} and \mathbf{p}_1 , which complies with the previous definition. For the final state leptons, in the dilepton rest frame and with $m_\ell = 0$, the expressions of the momenta are formally the same as in the $H \rightarrow Z\ell^+\ell^-$ case

$$\bar{p}_1 = \frac{m_Z}{2}(1, \sin \theta_1, 0, \cos \theta_1), \quad (72)$$

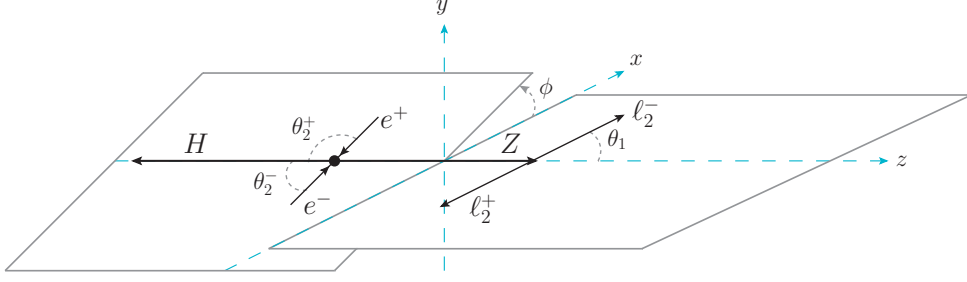


Figure 14: Kinematics for the scattering $e^+e^- \rightarrow HZ(\rightarrow \ell^+\ell^-)$.

$$\bar{p}_2 = \frac{m_Z}{2}(1, -\sin\theta_1, 0, -\cos\theta_1), \quad (73)$$

where again θ_1 is the angle between \mathbf{p}_1 , the momentum of ℓ^- , and the z axis.

With these definitions, the incoming momenta in the e^+e^- rest frame (denoted with an asterisk) are given by

$$p_-^* = \frac{\sqrt{q^2}}{2}(1, \sin\theta_2^- \cos\phi, \sin\theta_2^- \sin\phi, \cos\theta_2^-), \quad (74)$$

$$p_+^* = \frac{\sqrt{q^2}}{2}(1, -\sin\theta_2^- \cos\phi, -\sin\theta_2^- \sin\phi, -\cos\theta_2^-), \quad (75)$$

where, to make a clear distinction, the angle θ_2^- is the angle between the direction of flight of the e^- and the z axis in the e^+e^- rest frame. To best exploit the crossing symmetry of the two processes, one should describe the reaction using the angle θ_2^+ measured from the z axis to the direction of flight of the e^+ , since in $H \rightarrow Z\ell^+\ell^-$ we chose to use the angle between the direction of flight of ℓ_1^- and the z axis. Our results in Sec. 4 are therefore written in terms of the angle

$$\theta_2^+ \equiv \theta_2 = \pi - \theta_2^-, \quad (76)$$

which makes the expressions for the squared amplitude in decay and scattering formally identical.

B Explicit expressions for the J functions

Here we give the expressions of the J functions defined in Eq. (32). In the following results λ stands for

$$\lambda \equiv \lambda(1, s, r) = 1 + s^2 + r^2 - 2s - 2r - 2rs, \quad (77)$$

and we recall that $\kappa = 1 - s - r$. The couplings $g_{V,A}$ are those of Eq. (15) and contain the $d = 6$ corrections. The explicit expressions for the J functions read at $\mathcal{O}(1/\Lambda^2)$

$$J_1 = \frac{8\sqrt{2}m_H^2 G_F}{(s-r)^2} (g_A^2 + g_V^2)^2 r^3 s \times \left(1 + \frac{2(g_V^2 \hat{\alpha}_1^{\text{eff}} + g_A^2 \hat{\alpha}_2^{\text{eff}})}{g_A^2 + g_V^2} - \frac{2\kappa \hat{\alpha}_{ZZ}}{r} + \frac{g_V Q_\ell g_{\text{em}}(s-r)\kappa \hat{\alpha}_{AZ}}{(g_A^2 + g_V^2)rs} \right),$$

$$\begin{aligned}
J_2 &= \frac{4\sqrt{2}m_H^2 G_F}{(s-r)^2} (g_A^2 + g_V^2)^2 \kappa^2 r^2 \times \\
&\quad \left(1 + \frac{2(g_V^2 \hat{\alpha}_1^{\text{eff}} + g_A^2 \hat{\alpha}_2^{\text{eff}})}{g_A^2 + g_V^2} - \frac{8s \hat{\alpha}_{ZZ}}{\kappa} + \frac{4g_V Q_\ell g_{\text{em}}(s-r) \hat{\alpha}_{AZ}}{(g_A^2 + g_V^2) \kappa} \right), \\
J_3 &= -\frac{128\sqrt{2}m_H^2 G_F}{(s-r)^2} g_A^2 g_V^2 r^3 s \left(1 + \hat{\alpha}_1^{\text{eff}} + \hat{\alpha}_2^{\text{eff}} - \frac{2\kappa \hat{\alpha}_{ZZ}}{r} + \frac{Q_\ell g_{\text{em}}(s-r) \kappa \hat{\alpha}_{AZ}}{2g_V r s} \right), \\
J_4 &= -\frac{16\sqrt{2}m_H^2 G_F}{(s-r)^2} g_A^2 g_V^2 \kappa \sqrt{\frac{\lambda r^3}{s}} \left(4s \hat{\alpha}_{Z\tilde{Z}} + \frac{Q_\ell g_{\text{em}}(r-s) \hat{\alpha}_{A\tilde{Z}}}{g_V} \right), \\
J_5 &= \frac{\kappa}{4\sqrt{r s}} J_8, \\
J_6 &= -\frac{64\sqrt{2}m_H^2 G_F}{(s-r)^2} g_A^2 g_V^2 \kappa \sqrt{s r^5} \times \\
&\quad \left(1 + \hat{\alpha}_1^{\text{eff}} + \hat{\alpha}_2^{\text{eff}} + \frac{(\lambda - 2\kappa^2) \hat{\alpha}_{ZZ}}{r \kappa} + \frac{Q_\ell g_{\text{em}}(r-s)(\lambda - 2\kappa^2) \hat{\alpha}_{AZ}}{4g_V r s \kappa} \right), \\
J_7 &= \frac{4\sqrt{2}m_H^2 G_F}{(s-r)^2} (g_A^2 + g_V^2)^2 \kappa \sqrt{s r^5} \times \\
&\quad \left(1 + \frac{2(g_V^2 \hat{\alpha}_1^{\text{eff}} + g_A^2 \hat{\alpha}_2^{\text{eff}})}{g_A^2 + g_V^2} + \frac{(\lambda - 2\kappa^2) \hat{\alpha}_{ZZ}}{r \kappa} + \frac{g_V Q_\ell g_{\text{em}}(r-s)(\lambda - 2\kappa^2) \hat{\alpha}_{AZ}}{2(g_A^2 + g_V^2) r s \kappa} \right), \\
J_8 &= \frac{8\sqrt{2}m_H^2 G_F}{(s-r)^2} (g_A^2 + g_V^2)^2 r^2 \sqrt{\lambda} \left(2s \hat{\alpha}_{Z\tilde{Z}} + \frac{g_V Q_\ell g_{\text{em}}(r-s) \hat{\alpha}_{A\tilde{Z}}}{(g_A^2 + g_V^2)} \right), \\
J_9 &= J_1. \tag{78}
\end{aligned}$$

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